Unit 5 Sets

Exercise 5.1

1. **Which of the following collection is a set?**

(i) **A = The name of months in a year**

Solution: 'A' is a set because it is well-defined and names of all months are different.

(ii) **B = The prime factors of 12**

Solution: 'B' is a set because it is well-defined and its prime factors (1, 2, 3, 4, 6, 12) are different.

(iii) **C = Naughty boys in the classroom**

Solution: 'C' is not a set because it is not well-defined (we don't know about the particular students).

(iv) **D = The odd numbers between 9 and 19**

Solution: 'D' is a set because it is well-defined and the odd numbers between 9 and 19 are different.

(v) **E = Delicious food items of the city hotel**

Solution: 'E' is a set because it is well-defined (name is known) and food items are different.

(vi) **F = Vowels in the word of statistics**

Solution: 'F' is not a set because two elements are same (a, i, i).

(vii) **G = The letters of English alphabet**

Solution: 'G' is a set because it is well-defined and all English alphabet are different.

(viii) **H = The instruments in a geometry box**

Solution: 'H' is a set because it is well-defined and all instruments of geometry box are different.

(ix) **I = Set of rich persons**

Solution: 'I' is not a set because it is not well-defined (we don't know about the particular rich persons).

(x) **J = {a, b, c, d, e, a, b}**

Solution: 'J' is not a set because some elements are same.

(xi) **K= Sara, Aslam, Hamza, Ali, Aslam**

Solution: 'K' is not a set because two elements are same.

(xii) **L= The set of beautiful animals**

Solution: 'L' is not a set because it is not well-defined (we don't know about the particular animals).

> **Remember!**
> Any collection is said to be a set if it is well-defined and have different elements. To check whether a collection is a set or not check these two conditions.

2. **If N = {1, 2, 3, 4, ...} ,O = {1, 3, 5, 7, 9, 11} and E = {0, 2, 4, 6, 8, 10, 12, 14} then fill in the blanks using symbols $\in$ or $\notin$.**

(i) 100 ------- O
Solution: $100 \notin O$ because it is not an element of set O.

(ii) 14 ------- E
Solution: $14 \in E$ because it is contained in set E.

(iii) 27 ------- N
Solution: $27 \in N$ because it is contained in set N.

(iv) 19 ------- E
Solution: $19 \notin E$ because it is not an element of set E.

(v) 11 ------- O
Solution: $11 \in O$ because it is contained in set O.

(vi) 233 ------- O
Solution: $233 \notin O$ because it is not an element of set O.

(vii) **17 ------- E**
Solution: 17 $\notin$ E because it is not an element of set E.

(viii) **12 ------- E**
Solution: 12 $\in$ E because it is contained in set E.

(ix) **3 ------- E**
Solution: 3 $\notin$ E because it is not an element of set O.

(x) **4 ------- N**
Solution: 4 $\in$ N because it is contained in set N.

(xi) **98 ------- N**
Solution: 98 $\in$ N because it is contained in set N.

(xii) **300 ------- N**
Solution: 300 $\in$ N because it is contained in set N.

3. Write the following sets in descriptive form.

(i) A = {2, 4, 6, 8, 10, 12}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘A’ is a set of first six multiples of 2

(ii) D = {a, e, i, o, u}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘D’ is a set of vowels in English Alphabet

(iii) E = {5, 10, 15, 20, 25, 30}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘E’ is a set of multiples of first six multiples of 5

(iv) P = {1, 3, 5, 7, 9, 11, 13}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘P’ is a set of first seven odd numbers

(v) W = {0, 1, 2, 3, 4, 5, ... }

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘W’ is set of whole numbers

(vi) Z = {0, ±1, ±2, ±3, ±4, ...}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘Z’ is set of integers

(vii) L = {3, 6, 9, 12, 15, ...}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘L’ is a set of all multiples of 3

(viii) M = {1, 2, 3, 4, 5, ...}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = ‘M’ is set of natural numbers

4. Write the following sets in tabular form.

(i) A = The set of vowels in the word "education"

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = A = {e, u, a, i, o}

(ii) B = The set of even numbers less than 22

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = B = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

(iii) C = The set of the letters of the word "zain"

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = C = {z, a, i, n}

(iv) D = The set of the multiples of 4 less than 39

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = D = {4, 8, 12, 16, 20, 24, 28, 32, 36}

(v) E = The set of natural numbers

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = E = {1, 2, 3, 4, …}

(vi) F =The set of prime numbers between 9 and 27

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = F ={11, 13, 17, 19, 23}

(vii) G = The set of factors of 20

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = G = {1, 2, 4, 5, 10, 20}

(viii) H = The set of first 7 multiples of 6

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = H = {6, 12, 18, 24, 30, 36, 42}

Exercise 5.2

1. Separate the empty or singleton set from the following sets.

(i) The set of days of a week starting with 'F'.

Solution: First, find the elements of given set and then identify the type of set.
The set of days of a week starting with 'F' = {Friday}
It contains only one element so the given set is singleton set.

(ii) The set of even numbers between 22 and 26.
Solution: First, find the elements of given set and then identify the type of set.
The set of even numbers between 22 and 26 = {24}
It contains only one element so the given set is singleton set.

(iii) The set of horses having five legs.
Solution: First, find the elements of given set and then identify the type of set.
The set of horses having five legs = { }
It contains nothing so the given set is an empty set.

(iv) The set of capital city of Pakistan.

Solution: First, find the elements of given set and then identify the type of set.

The set of capital city of Pakistan = {Islamabad}

It contains only one element so the given set is singleton set.

(v) The set of moons on the Earth.

Solution: First, find the elements of given set and then identify the type of set.

The set of moons on the Earth = only one moon

It contains only one element so the given set is singleton set.

(vi) The set of odd numbers between 31 and 33.

Solution: First, find the elements of given set and then identify the type of set.

There is no odd number lies between 31 and 33.

It contains nothing so the given set is an empty set.

(vii) The set of natural numbers less than 1.

Solution: First, find the elements of given set and then identify the type of set.

Natural numbers start from 1 so there is no other natural number less than 1.

It contains nothing so the given set is an empty set.

(viii) The set of vowels starting with ‘L’.

Solution: First, find the elements of given set and then identify the type of set.

The set of vowels starting with ‘L’ = { }

It contains no element so the given set is an empty set.

(ix) The set of months of a year starting with 'O'.

Solution: First, find the elements of given set and then identify the type of set.

The set of months of a year starting with 'O' = {October}

It contains only one element so the given set is singleton set.

(x) The set of even numbers less than 0.

Solution: First, find the elements of given set and then identify the type of set.

Even numbers start from 0 so there is no other even number less than 0.

It contains nothing so the given set is an empty set.

2. Which of the following pair of sets are equal or equivalent sets? Also write them in symbolic form.

(i) A = {2, 4, 6, 9} and B = {2, 4, 9, 6}

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

Symbolic form: A = B

(ii) C = {a, b, c, d, e} and D = {a, c, d, e, b}

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

Symbolic form: C = D

(iii) X = {I, II, III, V} and Y = {III, II, I, V}

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

Symbolic form: X = Y

(iv) L = {a, b, c, d, e} and M = {1, 2, 3, 4, 5}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

Symbolic form: $L \leftrightarrow M$

(v) X = {orange, mango, apple, banana} and Y = {o, m, a, b}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

Symbolic form: $X \leftrightarrow Y$

(vi) Y = {table, book, pen, pencil} and X = {1, 2, 3, 4}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

Symbolic form: $Y \leftrightarrow X$

(vii) E = {chocolate, biscuit, fruit} and F = {February, March, April}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

Symbolic form: $E \leftrightarrow F$

(viii) P = {a, e, i, o, u} and Q = {u, o, i, a, e}

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

Symbolic form: $P = Q$

(ix) $Z = \{0, \pm1, \pm2, \pm3\}$, $S = \{0, 1, 2, 3, -1, -2, -3\}$

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

Symbolic form: $Z = S$

(x) $T = \{w, x, y, z\}$ and $B = \{+, \times, \div, -\}$

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

Symbolic form: $T \leftrightarrow B$

3. Separate the finite and infinite sets from the following sets.

(i) Set of stars in the universe.

Solution: Stars in the universe are uncountable so it is an infinite set.

(ii) Set of fingers in a hand.

Solution: Number of fingers in a hand are countable so it is a finite set.

(iii) Set of planets in our solar system.

Solution: Number of planets in our solar system are countable so it is a finite set.

(iv) Set of all prime numbers

Solution: Set of all prime numbers is not countable so it is an infinite set.

(v) Set of all even numbers.

Solution: Set of all even numbers is not countable so it is an infinite set.

(vi) Set of neutral integers.

Solution: It means {0} and it is countable so it is a finite set.

Exercise 5.3

1. Use '$\subset$' or '$\subseteq$' in the following blanks.

(i) {3, 5, 7}-------{1, 2, 3, ... , 10}

Solution: Here

$\{3, 5, 7\} \subset \{1, 2, 3, \ldots, 10\}$

(ii) { }------- {1, 2, 3, ... ,10}

Solution: Here

$\{\ \} \subset \{1, 2, 3, \ldots, 10\}$

(iii) {1}-------{1, 2, 3, ... , 10}

Solution: Here

$\{1\} \subset \{1, 2, 3, \ldots, 10\}$

(iv) {1, 2, 3,...,10}-----{1, 2, 3,..., 10}
Solution: Here
$\{1, 2, 3,...,10\} \subseteq \{1, 2, 3,..., 10\}$

(v) {2, 4, 6}-------{2, 4, 6}
Solution: Here
$\{2, 4, 6\} \subseteq \{2, 4, 6\}$

(vi) {0, 1, 2}-------{0, 1, 2, ...}
Solution: Here
$\{0, 1, 2\} \subset \{0, 1, 2, ...\}$

2. Find the total number of subsets of the following sets.

(i) {2, 3}
Solution: To calculate number of subsets use 2^n. Here, n = 2 so
$2^2 = 2 \times 2 = 4$

(ii) {−1, −2}
Solution: To calculate number of subsets use 2^n. Here, n = 2 so
$2^2 = 2 \times 2 = 4$

(iii) {2, 3, 5}
Solution: To calculate number of subsets use 2^n. Here, n = 3 so
$2^3 = 2 \times 2 \times 2 = 8$

(iv) {1, 5, 0}
Solution: To calculate number of subsets use 2^n. Here, n = 3 so
$2^3 = 2 \times 2 \times 2 = 8$

(v) {7, 9, 2, 3}
Solution: To calculate number of subsets use 2^n. Here, n = 4 so
$2^4 = 2 \times 2 \times 2 \times 2 = 16$

(vi) {−1, −2, −3, −4}
Solution: To calculate number of subsets use 2^n. Here, n = 4 so
$2^4 = 2 \times 2 \times 2 \times 2 = 16$

3. Find all possible subsets of the following sets.

(i) {1}
Solution: First, calculate total number of subsets using 2^n. Here, n = 1 so $2^1 = 2$.
Subsets of {1} = { }, {1}

(ii) {0}
Solution: First, calculate total number of subsets using 2^n. Here, n = 1 so $2^1 = 2$.
Subsets of {0} = { }, {0}

(iii) {0, 1}
Solution: First, calculate total number of subsets using 2^n. Here, n = 2 so $2^2 = 4$.
Subsets of {0, 1} = { }, {0}, {1}, {0, 1}

(iv) {+, −, ×}
Solution: First, calculate total number of subsets using 2^n. Here, n = 3 so $2^3 = 8$.
Subsets of {+, −, ×} = { }, {+}, {−}, {×}, {+, −}, {+, ×}, {−, ×}, {+, −, ×}

(v) {2, 7}
Solution: First, calculate total number of subsets using 2^n. Here, n = 2 so $2^2 = 4$.
Subsets of {2, 7} = { }, {2}, {7}, {2, 7}

(vi) {−1, −5, 7, 9}
Solution: First, calculate total number of subsets using 2^n. Here, n = 4 so $2^4 = 16$.
Subsets of {−1, −5, 7, 9} = { }, {−1}, {−5}, {7}, {9}, {−1, −5}, {−1, 7}, {−1, 9}, {−5, 7}, {−5, 9}, {7, 9}, {−1, −5, 7}, {−1, −5, 9}, {−1, 7, 9}, { −5, 7, 9}, {−1, −5, 7, 9}

4. Write 'Yes' on 'No' for the following statements.

(i) {1, 2, 3, ... , 30} can be universal set for {2, 4, 6, ... ,18}.
Solution: Yes. As all elements of {2, 4, 6, ... ,18} are contained in {1, 2, 3, ... , 30}.

(ii) {0, 1, 2, ... , 10} can be universal set for {5, 10, 15, 20}.
Solution: No. As all elements of {5, 10, 15, 20} are not contained in {0, 1, 2, ... , 10}.

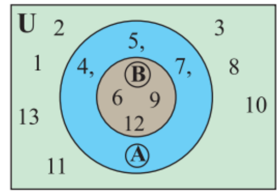
(iii) {2, 4, 6, ... , 80} can be universal set for {4, 8, 12, 16} and {8, 16, 24, 32, ... , 80}.
Solution: Yes. As all elements of {4, 8, 12, 16} and {8, 16, 24, 32, ... , 80} are contained in {2, 4, 6, ... , 80}.

5. **Draw the Venn diagram for the following sets.**

(i) U = {1, 2, 3, 4, ..., 12} A = {4, 5, 6, 7, 9, 12}; B = {6, 9,12}

Solution: To draw Venn diagram draw rectangle for universal set and circle for other subsets.

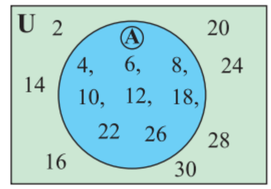
As all elements of 'B' contained in 'A' so we draw circle for 'B' inside the circle of 'A'.



(ii) U = {2, 4, 6, 8, 10, 12, 14, 16, ... , 30} ; A = {6, 8, 10, 12, 18, 22, 26}

Solution: To draw Venn diagram draw rectangle for universal set and circle for other subsets.

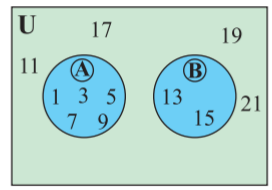
There is only one set so we draw only one circle.



(iii) U = {1, 3, 5, 7, 9, 11, 13, 15, ... , 21} ; A = {1, 3, 5, 7, 9} ; B = {13, 15}

Solution: To draw Venn diagram draw rectangle for universal set and circle for other subsets.

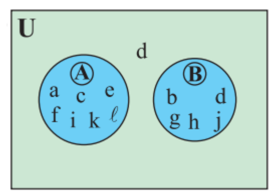
As 'A' and 'B' have no common terms so we draw two separate circles.



(iv) U = {a, b, c, d, e, f, g, h, i, j, k, *l*} A = {a, c, e , f, i, k, *l*} ;B = {b, d, g, h, j}

Solution: To draw Venn diagram draw rectangle for universal set and circle for other subsets.

As 'A' and 'B' have no common terms so we draw two separate circles.



Review Exercise 5

1. **Choose the correct option.**

(i) ϕ represents:

(a) subset (b) singleton set (c) improper set $\boxed{\text{(d) null set}}$

(ii) The set W = {0, 1, 2, 3, 4, ...} is:

(a) finite set (b) equal set $\boxed{\text{(c) infinite set}}$ (d) equivalent set

(iii) {Amina} is known as:

(a) proper set (b) null set (c) improper set $\boxed{\text{(d) singleton set}}$

(iv) When '0' is added in the set of natural numbers, then it becomes:

$\boxed{\text{(a) set of whole numbers}}$ (b) set of even numbers

(c) set of odd numbers (d) set of Integers

(v) Equal sets are always:

(a) proper sets (b) improper sets $\boxed{\text{(c) equivalent sets}}$ (d) not equal

(vi) In a set, an element of the set cannot appear more than:

$\boxed{\text{(a) once}}$ (b) two (c) three (d) four

(vii) Tabular form is also known as:

(a) descriptive form $\boxed{\text{(b) roster form}}$ (c) set-builder form (d) none of these

(viii) Two empty sets are always called:

(a) infinite sets (b) singleton sets $\boxed{\text{(c) equal sets}}$ (d) none of these

(ix) Which of the following is not a subset of {2, 4, 6}?

(a) {2,4, 6} (b) { } (c) {4, 6} $\boxed{\text{(d) \{0\}}}$

(x) Universal set is denoted by:

(a) A $\boxed{\text{(b) U}}$ (c) 0 (d) Z

2. Write the following sets in tabular form.

(i) A = The set of first four days of a week

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = A = {Monday, Tuesday, Wednesday, Thursday}

(ii) B = The set of first 15 natural numbers

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

(iii) C = The set of first five Solar months

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = C = {January, February, March, April, May}

(iv) D = The set of first five odd numbers

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = D = {1, 3, 5, 7, 9}

(v) E = The set of 8 numbers divisible by 8

Solution: To write the set in tabular form write the elements in curly brackets and separate them by commas.

Tabular form = E = {8, 16, 24, 32, 40, 48, 56, 64}

3. Write the following sets in descriptive form.

(i) A = {2, 4, 6, 8, ...}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = 'A' is the set of all multiples of 2 or set of even numbers

(ii) B = {2, 3, 5, 7, 11}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = 'B' is set of first five prime numbers

(iii) C = {−4, −3, −2, −1, 0 1, 2, 3, 4}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = 'C' is set of first five positive and negative integers

(iv) D = {31, 33, 35, 37, 39}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = 'D' is set of odd numbers greater than 30 and less than 40

(v) E = {a, e, i, o, u}

Solution: To write the given set in descriptive form just write it in words.

Descriptive form = 'E' is set of vowels in English Alphabet

4. Which of the following pairs of sets are equal or equivalent sets?

(i) A = {p, q, r}, B = {r, q, p}

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

(ii) C = {pen, book, room}, D = {1, 2, 3}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

(iii) E = {a, b, c, d, e, f, g, h}, F = {f, h, g, e, a, b, c, d}

Solution: As number of elements are equal and same in both sets so these sets are equal sets.

(iv) P = {blue, red, green}, Q = {Amina, Ali, Hamza}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

(v) R = {÷, ×, − , +}, S= {a, b, c, d}

Solution: As number of elements are equal but not same in both sets so these sets are equivalent sets.

5. If W = { 0, 1, 2, 3, 4, ... } , E ={2, 4, 6, 8, 10 } and P = { 1, 3, 5, 7, 9 }, then fill in the blanks by using the symbols $\in$ or $\notin$.

(i) 5 ------- W
Solution: 5 $\in$ W because it is contained in set W.

(ii) 12 ------- P
Solution: 12 $\notin$ P because it is not an element of set P.

(iii) 8 ------- E
Solution: 8 $\in$ E because it is contained in set E.

(iv) 9 ------- P
Solution: 9 $\in$ P because it is contained in set P.

(v) 0 ------- E
Solution: 0 $\notin$ E because it is not an element of set E.

(vi) 12 ------- W
Solution: 12 $\in$ W because it is contained in set W.

6. Find all the possible subsets of the following sets. Also separate proper and improper subsets.

(i) {2, 3}

Solution: First, calculate total number of subsets using 2^n. Here, n = 2 so $2^2 = 4$.

Subsets of {2, 3} = { }, {2}, {3}, {2, 3}

Set itself is improper subset and all other subsets are proper subsets.

(ii) {−1,−2}

Solution: First, calculate total number of subsets using 2^n. Here, n = 2 so $2^2 = 4$.

Subsets of {−1,−2} = { }, {−1}, {−2}, {−1,−2}

Set itself is improper subset and all other subsets are proper subsets.

(iii) {2, 3, 5}

Solution: First, calculate total number of subsets using 2^n. Here, n = 3 so $2^3 = 8$.

Subsets of {2, 3, 5} = { }, {2}, {3}, {5}, {2, 3},{2, 5}, {3, 5}, {2, 3, 5}

Set itself is improper subset and all other subsets are proper subsets.

(iv) {1, 5, 0}

Solution: First, calculate total number of subsets using 2^n. Here, n = 2 so $2^2 = 4$.
Subsets of {1, 5, 0} = { }, {1}, {5},{0},{1, 5},{1, 0},{5, 0},{1, 5, 0}

Set itself is improper subset and all other subsets are proper subsets.

(v) {7, 9, 2, 3}

Solution: First, calculate total number of subsets using 2^n. Here, n = 4 so $2^4 = 16$.
Subsets of {7, 9, 2, 3} = { }, {7}, {9}, {2}, {3}, {7, 9}, {7, 2}, {7, 3}, {9, 2}, {9, 3}, {2, 3}, {7, 9, 2}, {7, 9, 3}, {7, 2, 3}, { 9, 2, 3}, {7, 9, 2, 3}
Set itself is improper subset and all other subsets are proper subsets.

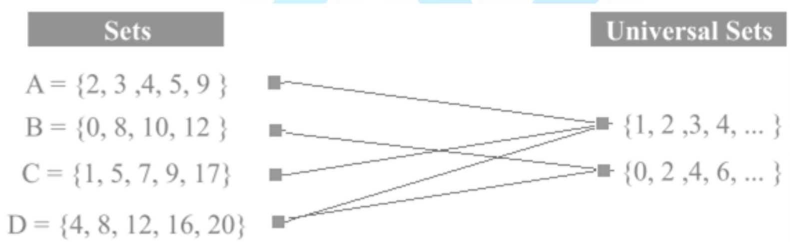
(vi) { −1,−2,−3,−4}

Solution: First, calculate total number of subsets using 2^n. Here, n = 4 so $2^4 = 16$.
Subsets of { −1,−2,−3,−4} = { }, { −1}, {−2}, { −3}, {−4}, { −1,−2}, { −1,−3}, { −1,−4}, { −2,−3}, {−2,−4},

{−3,−4}, { −1,−2,−3}, { −1,−2,−4}, { −1,−3,−4}, { −2,−3,−4}, { −1,−2,−3,−4}

Set itself is improper subset and all other subsets are proper subsets.

7. Match the given sets with respective universal set.
Solution: A set is called a universal set if it is a superset of all the sets under consideration. Any subset can have more than one universal set so it is not unique.

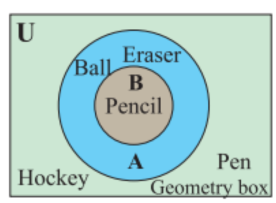


8. Draw Venn diagram for the following sets.

(i) U = {pen, hockey, ball, pencil, eraser, geometry box}; A= {ball, pencil, eraser}; B = {pencil}.

Solution: To draw Venn diagram draw rectangle for universal set and circle for other subsets.

As 'B' is subset of 'A' so we draw the circle for 'B' inside the circle of 'A'.



(ii) U= {1, 2, 3, 4, 5, 6, ..., 20}; A = {1, 2, 3, 4, 5, ..., 10} ; B = {11, 12, 13, ... , 20}.

Solution: To draw Venn diagram draw rectangle for universal set and circle for other subsets.

As 'A' and 'B' have no common terms so we draw two separate circles.