Unit 3 Rate, Ratio, Direct and Inverse Proportion

Exercise 3.1

1. Express each of the following ratio in its simplest form.

(i) $\frac{2}{3}:\frac{4}{5}$

Solution: To simplify given ratio, write it in the form of fraction.

$$\frac{2}{3}:\frac{4}{5}=\frac{\frac{2}{3}}{\frac{4}{5}}$$

$$=\frac{2}{3}\times\frac{5}{4_2}=\frac{1\times5}{3\times2}$$

$$=\frac{5}{6}$$

So, the simplest form is $5:6$.

(ii) $\frac{3}{7}:\frac{7}{3}$

Solution: To simplify given ratio, write it in the form of fraction.

$$\frac{3}{7}:\frac{7}{3}=\frac{\frac{3}{7}}{\frac{7}{3}}$$

$$=\frac{3}{7}\times\frac{3}{7}=\frac{3\times3}{7\times7}$$

$$=\frac{9}{49}$$

So, the simplest form is $9:49$.

(iii) $\frac{3}{4}:\frac{7}{9}$

Solution: To simplify given ratio, write it in the form of fraction.

$$\frac{3}{4}:\frac{7}{9}=\frac{\frac{3}{4}}{\frac{7}{9}}$$

$$=\frac{3}{4}\times\frac{9}{7}=\frac{3\times9}{4\times7}$$

$$=\frac{27}{28}$$

So, the simplest form is $27:28$.

(iv) **32 : 40**

Solution: To simplify given ratio, write it in the form of fraction.

$$32:40=\frac{32}{40}$$

Now, write the lowest form of fraction

$$\frac{32}{40}=\frac{32\div4}{40\div4}$$

$$=\frac{8}{10}$$

$$=\frac{8\div2}{10\div2}=\frac{4}{5}$$

So, the simplest form is $4:5$.

(v) **40 : 24**

Solution: To simplify given ratio, write it in the form of fraction.

$$40:24=\frac{40}{24}$$

Now, write the lowest form of fraction

$$\frac{40}{24}=\frac{40\div4}{24\div4}$$

$$=\frac{10}{6}$$

$$=\frac{10\div2}{6\div2}=\frac{5}{3}$$

So, the simplest form is $5:3$.

(vi) **35 : 42**

Solution: To simplify given ratio, write it in the form of fraction.

$$35:42=\frac{35}{42}$$

Now, write the lowest form of fraction

$$\frac{35}{42}=\frac{35\div7}{42\div7}$$

$$=\frac{5}{6}$$

So, the simplest form is $5:6$.

2. Express each of the following as a ratio of first quantity to the second. Also express in the lowest form.

(i) 80 *g*, 3 *kg*

Solution: First of all, make same units of given quantities.

1 *kg* = 1,000 *g* so 3 *kg* = 3,000 *g*

It implies

80 *g* to 3 *kg* = 80 *g* : 3,000 *g*

$$= \frac{80}{3000} = \frac{8^{2}}{300_{75}}$$

$$= \frac{2}{75}$$

$$= 2:75$$

So, the simplest form is 2 : 75.

(ii) 20 cm, 5 m

Solution: First of all, make same units of given quantities.

1 m = 100 cm so 5 m = 500 cm

It implies

20 cm to 5 m = 20 cm : 500 cm

$$= \frac{20}{500} = \frac{2^{1}}{50_{25}}$$

$$= \frac{1}{25}$$

$$= 1:25$$

So, the simplest form is 1 : 25.

(iii) 15 minutes, 3 hours

Solution: First of all, make same units of given quantities.

1 hour = 60 minutes so 3 hours = 180 minutes

It implies

15 minutes to 3 hours = 15 minutes : 180 minutes

$$= \frac{15}{180} = \frac{15^{1}}{180_{36_{12}}}$$

$$= \frac{1}{12}$$

$$= 1:12$$

So, the simplest form 1 : 12.

(iv) 5 hours, 50 minutes

Solution: First of all, make same units of given quantities.

1 hour = 60 minutes so 5 hours = 300 minutes

It implies

5 hours to 50 minutes = 300 minutes : 50 minutes

$$= \frac{300}{50} = \frac{300^{6}}{50_{1}}$$

$$= \frac{6}{1}$$

$$= 6:1$$

So, the simplest form 6 : 1.

(v) 18 kg , 12 kg

Solution: First of all, make same units of given quantities. Here units are already same so

18 kg to 12 *kg* = 18 kg : 12 kg

$$= \frac{18}{12} = \frac{18^{3}}{12_{2}}$$

$$= \frac{3}{2}$$

$$= 3:2$$

So, the simplest form is 3 : 2.

(vi) 100 m , 12 m

Solution: First of all, make same units of given quantities. Here units are already same so

100 m to 12 *m* = 100 m : 12 m

$$= \frac{100}{12} = \frac{100^{50^{25}}}{12_{6_{3}}}$$

$$= \frac{25}{3}$$

$$= 25:3$$

So, the simplest form is 25 : 3.

Remember!

Length	Mass	Capacity
1 kilometre = 1,000 metres 1 metre = 100 centimetre 1 centimetre = 10 millimetres	1 kilogram = 1,000 grams 1 gram = 1,000 milligrams	1 litre =1,000 millimetres

3. If a : b = 32 : 40 and b : c = 64 : 160. Find a : c.

Solution: Given that

$a:b=32:40$ $\qquad$ $b:c=64:160$

$\frac{a}{b}=\frac{32}{40}$ $\qquad$ $\frac{b}{c}=\frac{64}{160}$

As we know that

$$\frac{a}{b}\times\frac{b}{c}=\frac{a}{c}$$
$$\frac{32}{40}\times\frac{64}{160}=\frac{a}{c}$$
$$\frac{32\times64}{40\times160}=\frac{a}{c}$$
$$\frac{2048}{6400}=\frac{a}{c}$$
$$\frac{2048\div16}{6400\div16}=\frac{a}{c}$$
$$\frac{128}{400}=\frac{a}{c}$$
$$\frac{128\div16}{400\div16}=\frac{a}{c}$$
$$\frac{8}{25}=\frac{a}{c}$$

Hence, a : c = 8 : 25.

4. If a : b = $\frac{2}{3}:\frac{3}{4}$ and b : c = $\frac{4}{5}:\frac{5}{4}$. Find a : c.

Solution: Given that

$a:b=\frac{2}{3}:\frac{3}{4}$ $\qquad$ $b:c=\frac{4}{5}:\frac{5}{4}$

$\frac{a}{b}=\frac{2/3}{3/4}$ $\qquad$ $\frac{b}{c}=\frac{4/5}{5/4}$

$=\frac{2}{3}\times\frac{4}{3}$ $\qquad$ $=\frac{4}{5}\times\frac{4}{5}$

$=\frac{8}{9}$ $\qquad$ $=\frac{16}{25}$

As we know that

$$\frac{a}{b}\times\frac{b}{c}=\frac{a}{c}$$
$$\frac{8}{9}\times\frac{16}{25}=\frac{a}{c}$$
$$\frac{8\times16}{9\times25}=\frac{a}{c}$$
$$\frac{128}{225}=\frac{a}{c}$$

Hence, a : c = 128 : 225.

Exercise 3.2

1. In what ratio must 77 be increased to become 99.

Solution:

The required ratio = new value : old value

= 99 : 77

Divide both values by 11

= 9 : 7

2. In what ratio must 78 be decreased to become 52.

Solution:

The required ratio = new value : old value

= 52 : 78

Divide both values by 2

= 26 : 39

Divide both values by 13

= 2 : 3

3. In what ratio must 99 be increased to become 132.

Solution:

The required ratio = new value : old value

= 132 : 99

Divide both values by 3

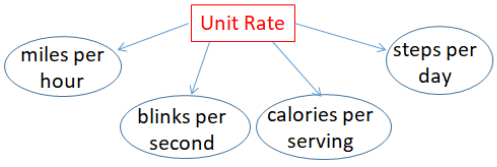
= 44 : 33

Divide both values by 11

= 4 : 3

Remember!

A unit rate is a ratio that compares two measurements (units) and the second measurement is 1 (per).



4. If 16 kg mangoes cost Rs. 5,600. Calculate the rate of mangoes.

Solution: The cost of 16 kg mangoes = Rs. 5,600

$$\text{The cost of 1 kg mangoes} = \frac{\text{Rs. } 5,600}{16}$$
$$= \text{Rs. } 350$$

So, the rate of mangoes is Rs. 350 per kg.

5. If Rs. 3,700 is charged for 148 units of electricity. Calculate the per unit average rate of electricity.

Solution: The cost of 148 units of electricity = Rs. 3,700

$$\text{The cost of 1 unit of electricity} = \frac{\text{Rs. } 3,700}{148}$$
$$= \text{Rs. } 25$$

So, the rate of electricity is Rs. 25 per unit.

6. If a car uses 50 litres of petrol to travel 600 *km*. Calculate the average rate of consumption of petrol.

Solution: The car travels in 50 litres of petrol = 600 *km*

$$\text{The car travels in 1 litre of petrol} = \frac{600\,km}{50}$$
$$= 12\,km$$

So, the average rate of consumption of petrol is 12 *km* per litre.

Exercise 3.3

1. Find the value of "a" in the following proportions.

(i) 14 : 5 : : a : 35

Solution: As we know that

Product of means = Product of extremes

$5 \times a = 14 \times 35$

Divide both sides by 5

$$\frac{5 \times a}{5} = \frac{14 \times 35^{7}}{5}$$
$$a = 14 \times 7$$
$$a = 98$$

Hence, the value of 'a' is 98.

(ii) 2 : 3 : : a : 27

Solution: As we know that

Product of means = Product of extremes

$3 \times a = 2 \times 27$

Divide both sides by 3

$$\frac{3 \times a}{3} = \frac{2 \times 27^{9}}{3}$$
$$a = 2 \times 9$$
$$a = 18$$

Hence, the value of 'a' is 18.

(iii) 7 : 9 : : a : 81

Solution: As we know that

Product of means = Product of extremes

$9 \times a = 7 \times 81$

Divide both sides by 9

$$\frac{9 \times a}{9} = \frac{7 \times 81^{9}}{9}$$
$$a = 7 \times 9$$
$$a = 63$$

Hence, the value of 'a' is 63.

(iv) 13 : 17 : : a : 34

Solution: As we know that

Product of means = Product of extremes

$17 \times a = 13 \times 34$

Divide both sides by 17

$$\frac{17 \times a}{17} = \frac{13 \times 34^{2}}{17}$$
$$a = 13 \times 2$$
$$a = 26$$

Hence, the value of 'a' is 26.

2. 10 people can dig a well in 35 days. In how many days will 7 people dig the well?

Solution: People and days are inversely proportional to each other.

People	::	Days
↓ 10		35 ↑
7		x

For inverse proportion use arrows in different direction

$$\frac{10}{7} = \frac{x}{35}$$

Apply cross multiplication

$$10 \times 35 = x \times 7$$

Divide both sides by 7

$$\frac{10 \times 35}{7} = \frac{x \times \cancel{7}}{\cancel{7}}$$

$$\frac{350}{7} = x$$

$$50 = x$$

Hence, 7 people will dig the well in 50 days.

Remember!

X increase	→	Y increase	(Direct proportion)
X decrease	→	Y decrease	(Direct proportion)
X increase	→	Y decrease	(Inverse proportion)
X decrease	→	Y increase	(Inverse proportion)

3. Total cost of 35 lottery tickets is Rs. 2,100. Find the cost of 25 lottery tickets.

Solution: Cost and tickets are directly proportional to each other.

Cost	::	Tickets
↑ 2100		35 ↑
x		25

For direct proportion use arrows in same direction

$$\frac{x}{2100} = \frac{25}{35}$$

Apply cross multiplication

$$x \times 35 = 25 \times 2100$$

Divide both sides by 35

$$\frac{x \times 35}{35} = \frac{25 \times 2100}{35}$$

$$x = \frac{52500}{35}$$

$$x = 1500$$

Hence, the cost of 25 lottery tickets is Rs. 1,500.

4. The cost of 7 metres of cloth is Rs. 2,100. How many metres of the cloth can be purchased by Rs. 27,000?

Solution: Cost and length are directly proportional to each other.

Cost	::	Length
↑ 2,100		7 ↑
27,000		x

For direct proportion use arrows in same direction

$$\frac{270\cancel{00}}{21\cancel{00}} = \frac{x}{7}$$

Apply cross multiplication

$$270 \times 7 = x \times 21$$

Divide both sides by 21

$$\frac{1890}{21} = \frac{x \times 21}{21}$$

$$90 = x$$

Hence, 90 *m* of cloth can be purchased by Rs. 27,000.

5. The cost of 200 *kg* rice is Rs. 40,000. How many kilograms of rice can be purchased for Rs. 10,000?

Solution: Cost and weight are directly proportional to each other.

Cost	::	Weight
40,000		200
10,000		x

For direct proportion use arrows in same direction

$$\frac{10\cancel{,000}}{40\cancel{,000}} = \frac{x}{200}$$

$$\frac{10}{40} = \frac{x}{200}$$

Apply cross multiplication

$$10 \times 200 = x \times 40$$

Divide both sides by 40

$$\frac{10 \times 200}{40} = \frac{x \times 40}{40}$$

$$\frac{2000}{40} = x$$

$$50 = x$$

Hence, 50 *kg* of rice can be purchased by Rs. 10,000.

6. A bus travels 15 km in 6 litres. How far would it travel in 72 litres?

Solution: Distance and fuel are directly proportional to each other.

Distance	::	Fuel
15		6
x		72

For direct proportion use arrows in same direction

$$\frac{x}{15} = \frac{72}{6}$$

Apply cross multiplication

$$x \times 6 = 72 \times 15$$

Divide both sides by 6

$$\frac{x \times 6}{6} = \frac{72 \times 15}{6}$$

$$x = \frac{1080}{6}$$

$$x = 180$$

Hence, the bus will travel 180 *km* in 72 litres.

7. If the weight of 11 sheets of paper is 121 grams, how many sheets of the same paper would weight 770 grams?

Solution: Number of sheets and weight are directly proportional to each other.

Sheets	::	Weight
11		121
x		770

For direct proportion use arrows in same direction

$$\frac{x}{11}=\frac{770}{121}$$

Apply cross multiplication

$$x\times 121 = 770\times 11$$

Divide both sides by 121

$$\frac{x\times 121}{121}=\frac{770\times 11}{121}$$

$$x=\frac{8470}{121}$$

$$x = 70$$

Hence, 70 sheets of paper would weigh 770 grams.

8. If 16 men can do a job in 40 days, how many men will do the same job in 32 days?

Solution: Men and days are inversely proportional to each other.

Men	::	Days
16		40
x		32

For inverse proportion use arrows in different direction

$$\frac{x}{16}=\frac{40}{32}$$

Apply cross multiplication

$$x\times 32 = 40\times 16$$

Divide both sides by 32

$$\frac{x\times 32}{32}=\frac{40\times 16}{32}$$

$$x=\frac{640}{32}$$

$$x = 20$$

Hence, 20 men will do the same job in 32 days.

Review Exercise 3

1. Choose the correct option.

(i) The value of "a" in the proportion 5 : 10 :: a :100 is:

(a) 40 (b) 50 (c) 60 (d) 80

(ii) In what ratio must 45 be increased to become 75?

(a) 3 : 5 (b) 5 : 3 (c) 7 : 3 (d) 3 : 7

(iii) In what ratio must 150 be decreased to become 100?

(a) 2 : 3 (b) 3 : 2 (c) 5 : 3 (d) 1 : 2

(iv) The simplest firm of the ratio $\frac{3}{4}:\frac{4}{5}$ is:

(a) 15 : 16 (b) 5 : 4 (c) 3 : 4 (d) 1 : 2

(v) The ratio of 400 g to 3 kg is:

(a) 400 : 3 (b) 4 : 3 (c) 2 : 15 (d) 1 : 12

2. If a : b = 12 : 20 and b : c = 30 : 50. Find a : c.

Solution: Given that

$$a:b=12:20 \qquad b:c=30:50$$

$$\frac{a}{b}=\frac{\cancel{12}^{3}}{\cancel{20}_{5}} \qquad \frac{b}{c}=\frac{3\cancel{0}}{5\cancel{0}}$$

$$\frac{a}{b}=\frac{3}{5} \qquad \frac{b}{c}=\frac{3}{5}$$

As we know that

$$\frac{a}{b}\times\frac{b}{c}=\frac{a}{c}$$

$$\frac{3}{5}\times\frac{3}{5}=\frac{a}{c}$$

$$\frac{9}{25}=\frac{a}{c}$$

Hence, a : c = 9 : 25.

3. In what ratio 66 be increased to become 99?

Solution:

The required ratio = new value : old value

= 99 : 66

Divide both values by 11

= 9 : 6

Divide both values by 3

= 3 : 2

4. In what ratio 86 be decreased to become 43?

Solution:

The required ratio = new value : old value

= 43 : 86

Divide both values by 43

= 1 : 2

5. If 15 kg apples cost Rs. 3,000. Calculate the rate of apples.

Solution:

The cost of 15 kg apples = Rs. 3,000

$$\text{The cost of 1 kg apples} = \frac{\text{Rs. } 3,000}{15}$$

$$= \text{Rs. } 200$$

So, the rate of apples is Rs. 200 per kg.

6. Find the value of "a" in the following proportions.

(i) 16 : 5 :: a : 35

Solution: As we know that

Product of means = Product of extremes

$$5 \times a = 16 \times 35$$

Divide both sides by 5

$$\frac{5 \times a}{5} = \frac{16 \times 35^{7}}{5}$$

$$a = 16 \times 7$$

$$a = 112$$

Hence, the value of 'a' is 112.

(ii) 8 : 9 :: a : 81

Solution: As we know that

Product of means = Product of extremes

$$9 \times a = 8 \times 81$$

Divide both sides by 9

$$\frac{9 \times a}{9} = \frac{8 \times 81^{9}}{9}$$

$$a = 8 \times 9$$

$$a = 72$$

Hence, the value of 'a' is 72.

(iii) 11: 17 :: a : 34

Solution: As we know that

Product of means = Product of extremes

$$17 \times a = 11 \times 34$$

Divide both sides by 17

$$\frac{17 \times a}{17} = \frac{11 \times 34^{2}}{17}$$

$$a = 11 \times 2$$

$$a = 22$$

Hence, the value of 'a' is 22.

7. If 20 men can do a job in 40 days. How many men will do the same job in 80 days?

Solution: Men and days are inversely proportional to each other.

People	::	Days
↓ 20		40 ↑
x		80

For inverse proportion use arrows in different direction

$$\frac{20}{x} = \frac{80}{40}$$

Apply cross multiplication

$$20 \times 40 = x \times 80$$

Divide both sides by 80

$$\frac{20 \times 40}{80} = \frac{x \times 80}{80}$$

$$\frac{800}{80} = x$$

$$10 = x$$

Hence, 10 men will do the same job in 80 days.

8. A car travels 10 *km* in 3 litres. How far would it travel in 60 litres?

Solution: Distance and fuel are directly proportional to each other.

Distance	::	Fuel
10		3
x		60

For direct proportion use arrows in same direction

$$\frac{x}{10} = \frac{60}{3}$$

Apply cross multiplication

$$x \times 3 = 60 \times 10$$

Divide both sides by 3

$$\frac{x \times 3}{3} = \frac{60 \times 10}{3}$$

$$x = \frac{600}{3}$$

$$x = 200$$

Hence, the car will travel 200 *km* in 60 litres.