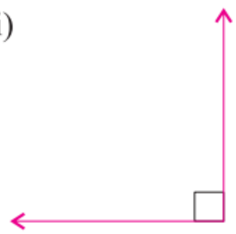
Unit 13 Geometry

Exercise 13.1

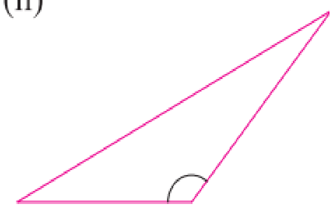
1. Identify which of the following angles are acute, obtuse, right, reflex and straight?

(i)



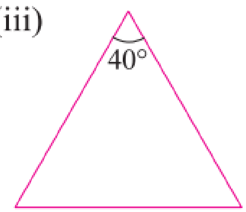
Solution: Given angle shows right angle because two lines intersect each other perpendicularly. You can identify it by small square.

(ii)



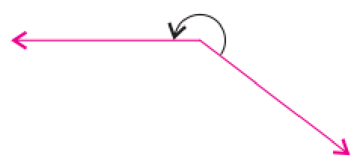
Solution: Given angle shows obtuse angle because it is more than 90^o. You can identify it by amount of turn in its arm.

(iii)



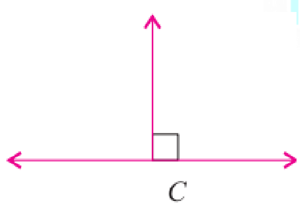
Solution: Given angle shows acute angle because it is less than 90^o.

(iv)



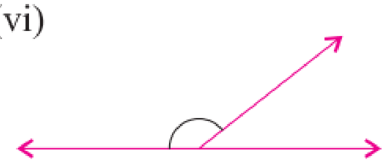
Solution: Given angle shows reflex angle because it is on the outside of arms. It is greater than 180^o.

(v)



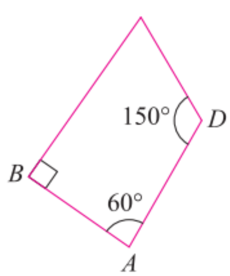
Solution: Given angle shows right angle because two lines intersect each other perpendicularly. You can identify it by small square.

(vi)



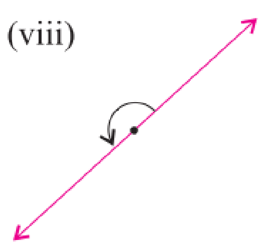
Solution: Given angle shows obtuse angle because it is more than 90^o. You can identify it by amount of turn in its arm.

(vii)



Solution: Given angle B shows right angle because two lines intersect each other perpendicularly. You can identify it by small square.

(viii)



Solution: Given angle shows straight angle because it is made on the straight line. Straight angle means angle of measure 180^o.

2. Which of the following pairs of angles are complementary and which are supplementary?

(i) 25^o, 65^o

Solution: This pair shows complementary angles because sum of both angles is 90^o.

$$25^o + 65^o = 90^o$$

(ii) 40^o, 140^o

Solution: This pair shows supplementary angles because sum of both angles is 180^o.

$$40^o + 140^o = 180^o$$

(iii) 79^o, 101^o

Solution: This pair shows supplementary angles because sum of both angles is 180^o.

$$79^o + 101^o = 180^o$$

(iv) 38.5^o, 51.5^o

Solution: This pair shows complementary angles because sum of both angles is 90^o.

$$38.5^o + 51.5^o = 90^o$$

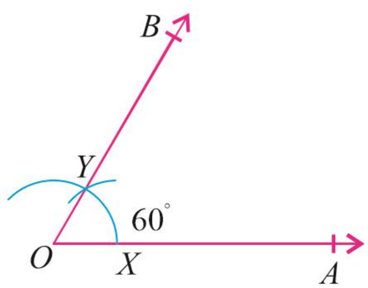
(v) 0^o, 180^o

Solution: This pair shows supplementary angles because sum of both angles is 180^o.

$$0^o + 180^o = 180^o$$

3. Use compass and ruler to construct the following angles:

(i) 60^o



Solution: To construct an angle of 60^o using compass and ruler follow the mentioned steps.

Step 1: Draw a ray $\overrightarrow{OA}$ of suitable length.

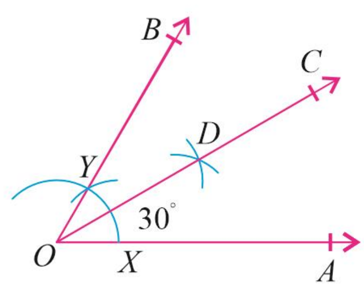
Step 2: Open the compass and draw an arc of suitable radius which cuts ray $\overrightarrow{OA}$ at point X.

Step 3: Draw another arc of same radius whose center is X which cuts previous arc at point Y.

Step 4: Draw a ray $\overrightarrow{OB}$ through Y using ruler.

$$m\angle AOB = 60^o$$

(ii) 30^o



Solution: To construct an angle of 30^o using compass and ruler follow the mentioned steps.

Step 1: Draw a ray $\overrightarrow{OA}$ of suitable length.

Step 2: Open the compass and draw an arc of suitable radius which cuts ray $\overrightarrow{OA}$ at point X.

Step 3: Draw another arc of same radius whose center is X which cuts previous arc at point Y. It show angle of 60^o.

Step 4: Now make an angle bisector using point X and Y.

Step 5: Draw an arc using point X and other arc using point Y. Both arcs cut each other at point D.

Step 6: Draw a ray $\overrightarrow{OC}$ through D using ruler.

$$m\angle AOC = 30^o$$

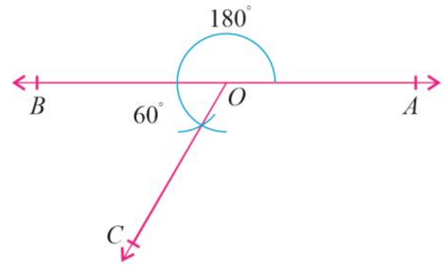
(iii) 240°

Solution: To construct an angle of 240° using compass and ruler follow the mentioned steps.

Step 1: Draw a line $\overleftrightarrow{AB}$ of suitable length. As straight line shows angle of 180°.

$$240^o = 180^o + 60^o$$

It means, we have to make extra angle of 60° with straight angle.



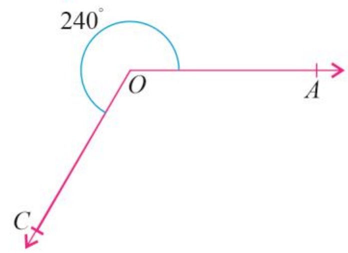
Step 2: Open the compass and draw an arc of suitable radius which cuts ray $\overrightarrow{OB}$.

Step 3: Draw another arc of same radius which cuts previous arc. It show angle of 60°.

Step 4: Draw a ray $\overrightarrow{OC}$ through O using ruler.

Step 5: Now join the angle outside which will be the measure of 240°.

$$m\angle AOC = 240^o$$



(iv) 90°

Solution: To construct an angle of 90° using compass and ruler follow the mentioned steps.

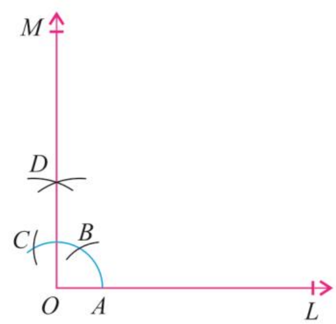
Step 1: Draw a ray $\overrightarrow{OL}$ of suitable length.

Step 2: Open the compass and draw an arc of suitable radius which cuts ray $\overrightarrow{OL}$ at point A.

Step 3: Draw another arc of same radius whose center is A which cuts previous arc at point B, this arc show 60°.

Step 4: Draw another arc of same radius whose center is B which cuts previous arc at point C, this arc shows 120°.

Step 5: Draw two arcs with center at B and C which cut each other at point D. It is the bisector of the angle means 30°.



Step 6: Draw a ray $\overrightarrow{OM}$ through point D using ruler.

$m\angle LOM = 90^o$

4. Construct a right angled triangle ABC in which $m\angle A = 60^o$, $m\overline{AC} = 7.2$

Solution: To construct a right angled triangle follow the mentioned steps.

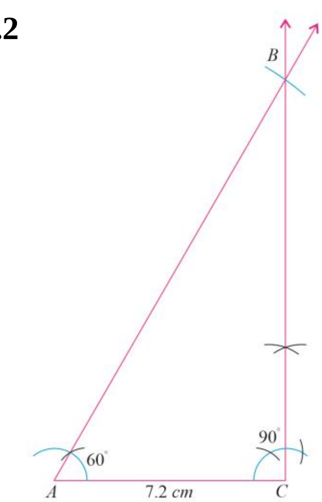
Step 1: Draw a line segment $m\overline{AC} = 7.2\ cm$.

Step 2: Using compass and ruler draw angle of 90° at point C.

Step 3: Using compass and ruler draw angle of 60° at point A.

Step 4: Extend arm of angle A such that it cuts the arm of angle C at point B.

Hence, ΔABC is the required right angled triangle.



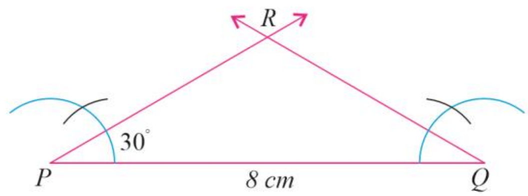
5. Construct an isosceles triangle given that $m\angle P = 30^o$, $m\overline{PQ} = 8$ cm and $m\overline{PR} = m\overline{QR}$.

Solution: To construct an isosceles triangle follow the mentioned steps.

Step 1: Draw a line segment $m\overline{PQ} = 8\ cm$.

Step 2: Using compass and ruler draw angle of 30^o at point P.

Here $m\overline{PR} = m\overline{QR}$ as length is not mentioned so we can draw angle of 30 at point Q.



Step 3: Using compass and ruler draw angle of 30^o at point Q.

Step 4: Where arms of both angle cut each other write the name R of that point.

Hence, ΔPQR is the required isosceles triangle.

6. Construct an equilateral ΔLMN given that $m\overline{LM} = 6.8$ cm.

Solution: To construct an equilateral triangle follow the mentioned steps.

Step 1: Draw a line segment $m\overline{LM} = 6.8\ cm$.

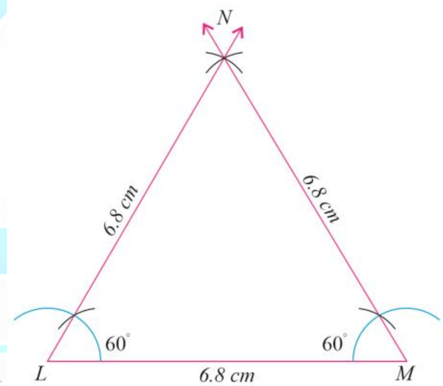
In equilateral triangle all sides are equal and all angles are of measure 60^o.

Step 2: Using compass and ruler draw angle of 60^o at point L.

Step 3: Using compass and ruler draw angle of 60^o at point M.

Step 4: Where arms of both angles cut each other, write the name N of that point.

Hence, ΔLMN is the required equilateral triangle.



7. Construct a triangle DEF such that $m\angle D = 45^o$, $m\angle F = 60^o$ and $m\overline{DF} = 9$ cm.

Solution: To construct the required triangle follow the mentioned steps.

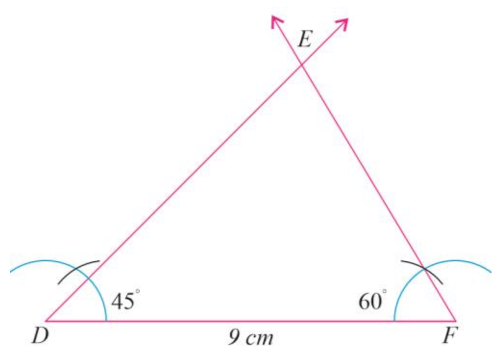
Step 1: Draw a line segment $m\overline{DF} = 9\ cm$.

Step 2: Using compass and ruler draw angle of 45^o at point D.

Step 3: Using compass and ruler draw angle of 60^o at point F.

Step 4: Where arms of both angles cut each other write the name E of that point.

Hence, ΔDEF is the required triangle.



8. Construct a triangle EFG such that $m\angle E = 70^o$, $m\overline{EF} = 7.8$ cm and $m\overline{FG} = 8.3$ cm.

Solution: To construct the required triangle follow the mentioned steps.

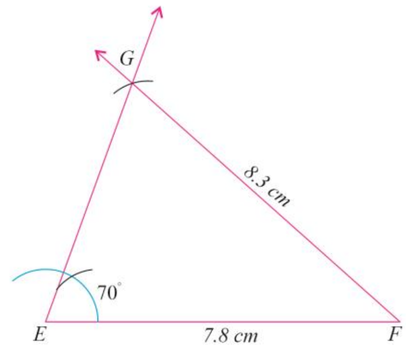
Step 1: Draw a line segment $m\overline{EF} = 7.8\ cm$.

Step 2: Using compass and ruler draw angle of 70^o at point E and extend the arm.

Step 3: Using compass draw an arc of radius 8.3 cm whose center is F which cut the arm at point G.

Step 4: Join the point F and G using ruler.

Hence, ΔEFG is the required triangle.



9. Construct a right angled triangle in which hypotenuse is 10 cm and other two sides have lengths 8 cm and 6 cm.

Solution: To construct a right angled triangle follow the mentioned steps.

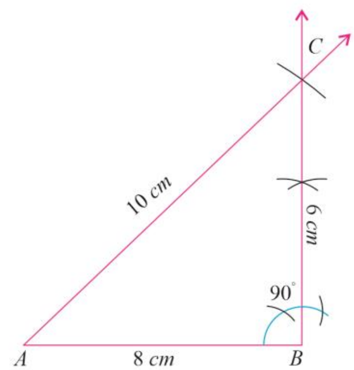
Step 1: Draw a line segment $m\overline{AB} = 8\ cm$.

Step 2: Using compass and ruler draw angle of 90^o at point B and extend the arm.

Step 3: Using compass draw an arc of radius 6 cm whose center is B which cut the arm at point C.

Step 4: Join the point A and C using ruler.

Hence, ΔABC is the required right angled triangle.



10. Construct a right angled triangle such that base length is 6.5 cm and angle at that base is 75^o.

Solution: To construct a right angled triangle follow the mentioned steps.

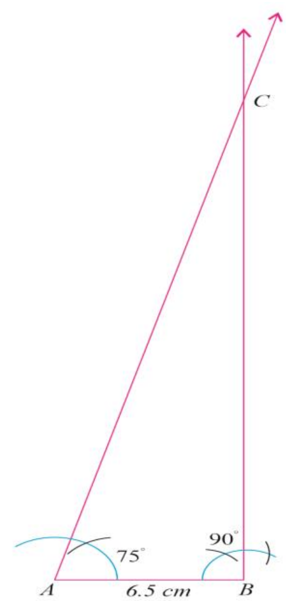
Step 1: Draw a line segment $m\overline{AB} = 6.5\ cm$.

Step 2: Using compass and ruler draw angle of 75^o at point A and extend the arm.

Step 3: Using compass and ruler draw angle of 90^o at point B and extend the arm.

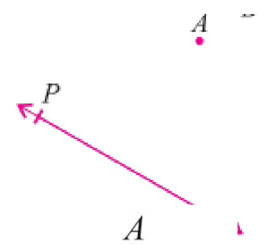
Step 4: Where arms of both angles cut each other write the name C of that point.

Hence, ΔABC is the required right angled triangle.



Exercise 13.2

1. Draw a perpendicular from a point A to the line PQ of length 7 cm.



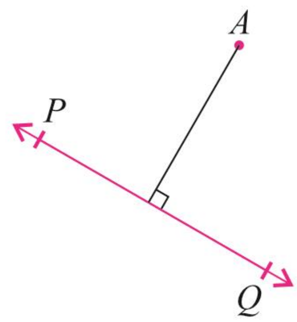
Solution: To draw a perpendicular from a point on a line follow the mentioned steps.

Step 1: Using compass draw an arc whose center is point A which intersect the line PQ at point L and M.

Step 2: Draw two arcs with center at point A and Point B of suitable radius intersecting at point T.

Step 3: Draw a ray from P passing through the intersecting arcs.

Hence, the line from dot A is perpendicular on the line PQ.

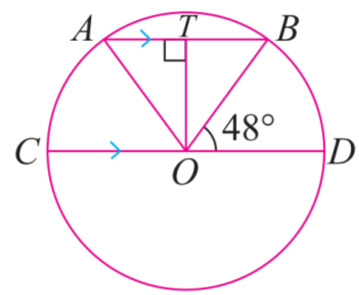


2. $\overline{AB}$ is parallel to $\overline{CD}$

(i) Find (a) $m\angle OBT$

(b) $m\angle OAT$

(ii) Is $\overline{OT}$ perpendicular to both $\overline{AB}$ and $\overline{CD}$?



Solution: Given that $\overline{AB}$ is parallel to $\overline{CD}$

(i) To find the given angles

(a) Vertically opposite angles are always equal. Here angle BOD and angle OBT are vertically opposite so both are equal. It means

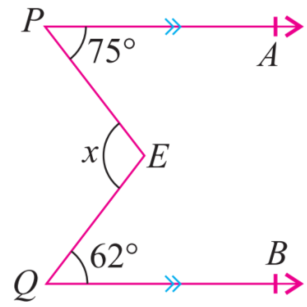
$$m\angle OBT = 48^o$$

(b) As line $\overline{TO}$ is perpendicular on line segment CD. It means the line TO makes angle of 90°. Same as angle BOD the measure of angle AOC is also 48°. Vertically opposite angles are always equal. Here angle AOC and angle OAT are vertically opposite so both are equal. It means

$$m\angle OAT = 48^o$$

(ii) We know that, a line is said to be perpendicular on another line if it makes angle of 90° with it. The line $\overline{OT}$ is perpendicular on both line segments $\overline{AB}$ and $\overline{CD}$.

3. $\overrightarrow{PA}$ and $\overrightarrow{QB}$ are parallel. Find the value of *x*.



Solution: Given that $\overrightarrow{PA}$ and $\overrightarrow{QB}$ are parallel.

Let's complete the triangle PQE by drawing a line.

If we observe, the line $\overline{PQ}$ is perpendicular on the rays $\overrightarrow{PA}$ and $\overrightarrow{QB}$.

Let suppose the internal angles of triangle PQE are '*a*', '*b*' and *x*.

Here, $a + 75^o = 90^o$ and $b + 62^o = 90^o$

It means, $a = 90^o - 75^o = 15^o$

$b = 90^o - 62^o = 28^o$

Now, to find value of '*x*' use the property:

The sum of all internal angles of a triangle is equal to 180°.

So, $15^o + 28^o + x = 180^o$

$43^o + x = 180^o$

$x = 180^o - 43^o$

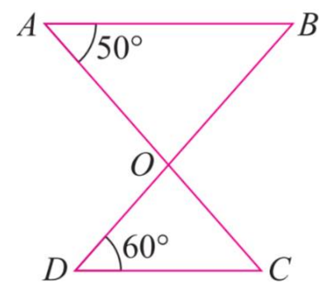
$x = 137^o$

4. (i) Which angles are vertically opposite to each other?

(ii) Find (a) $m\angle OCD$

(b) $m\angle DOC$

(iii) Which pairs of angles are alternate?



Solution: In the given figure

(i) $m\angle AOB$ and $m\angle DOC$ are vertically opposite.

(ii) To find the required angle apply the definition of alternate angles.

Here $m\angle OAB$ and $m\angle OCD$ are alternate angles so both are equal in measure. It means:

(a) $m\angle OCD = 50^o$

Now, to calculate second angle use the property:

The sum of all internal angles of a triangle is equal to 180^o.

(b) $m\angle DOC + 60^o + 50^o = 180^o$

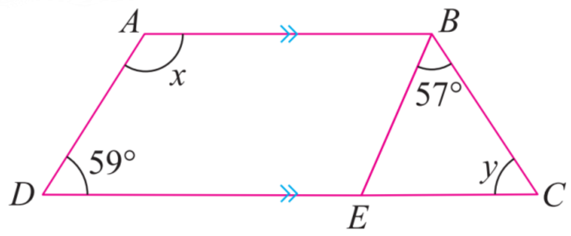
$m\angle DOC + 110^o = 180^o$

$m\angle DOC = 180^o - 110^o$

$m\angle DOC = 70^o$

(iii) Here $m\angle OAB$ and $m\angle OCD$, $m\angle ODC$ and $m\angle OBA$ are alternate angles.

5. ABCD is a trapezium and ABED is a parallelogram. Find *x* and *y*.



Solution: In the given figure, ABCD is a trapezium and ABED is a parallelogram.

In parallelogram ABED, the angles on the same side of the transversal are supplementary, that means they add up to 180^o.

It means, $x + 59^o = 180^o$

Which implies $x = 180^o - 59^o$

$x = 121^o$

In Triangle BEC the angle at vertex E will be $180^o - 121^o = 59^o$

It means, $y + 57^o + 59^o = 180^o$

Which implies $y + 116^o = 180^o$

$y = 180^o - 116^o$

$y = 64^o$

Exercise 13.3

1. Find interior angles of a regular polygon with *n* sides for the given number of sides:

(i) $n = 8$

Solution: A regular polygon has all its interior angles equal in measure. The sum of all interior angles of a regular polygon can be found by using the formula:

Sum of interior angles of regular polygon with *n* sides = $(n - 2) \times 180^o$

Here, n = 8

Sum of interior angles of regular polygon with 8 sides = $(8 - 2) \times 180^o$

$= 6 \times 180^o$

$= 1080^o$

To find measure of each interior angle, divide the sum of interior angles by number of sides

Therefore, measure of each interior angle = $\frac{1080^o}{8} = 135^o$

(ii) $n = 16$

Solution: A regular polygon has all its interior angles equal in measure. The sum of all interior angles of a regular polygon can be found by using the formula:

Sum of interior angles of regular polygon with *n* sides = $(n - 2) \times 180^o$

Here, n = 16

Sum of interior angles of regular polygon with 16 sides = $(16 - 2) \times 180^o$

$= 14 \times 180^o$

$= 2520^o$

To find measure of each interior angle, divide the sum of interior angles by number of sides.

Therefore, measure of each interior angle = $\frac{2520^o}{16} = 157.5^o$

(iii) $n = 18$

Solution: A regular polygon has all its interior angles equal in measure. The sum of all interior angles of a regular polygon can be found by using the formula:

Sum of interior angles of regular polygon with *n* sides = $(n - 2) \times 180^o$

Here, n = 18

Sum of interior angles of regular polygon with 18 sides = $(18 - 2) \times 180^o$

$= 16 \times 180^o$

$= 2880^o$

To find measure of each interior angle, divide the sum of interior angles by number of sides.

Therefore, measure of each interior angle = $\frac{2880^o}{18} = 160^o$

2. Find exterior angles of a regular polygon with number of sides:

(i) 10

Solution: The sum of all exterior angles of a regular polygon is equal to 360^o. First of all find the measure of an interior angle.

Step 1: Find an interior angle of the regular polygon with *n* sides using the formula:

$$\frac{(n-2)\times 180^o}{n}$$

Here, n = 10

The measure of each interior angle of a regular polygon with 10 sides $= \frac{(10-2)\times 180^o}{10}$

$= \frac{8\times 180^o}{10}$

$= \frac{1440^o}{10}$

$= 144^o$

Step 2: Find measure of each exterior angle, subtract obtained interior angle from 180^o because each interior angle with adjacent exterior angle make angle of 180^o. So,

Measure of each exterior angle $= 180^o - 144^o$

$= 26^o$

(ii) 12

Solution: The sum of all exterior angles of a regular polygon is equal to 360^o. First of all find the measure of an interior angle.

Step 1: Find an interior angle of the regular polygon with *n* sides using the formula:

$$\frac{(n-2)\times 180^o}{n}$$

Here, n = 12

The measure of each interior angle of a regular polygon with 12 sides $= \frac{(12-2)\times 180^o}{12}$

$= \frac{10\times 180^o}{12}$

$= \frac{1800^o}{12}$

$= 150^o$

Step 2: Find measure of each exterior angle, subtract obtained interior angle from 180^o because each interior angle with adjacent exterior angle make angle of 180^o. So,

Measure of each exterior angle $= 180^o - 150^o$

$= 30^o$

(iii) 18

Solution: The sum of all exterior angles of a regular polygon is equal to 360^o. First of all find the measure of an interior angle.

Step 1: Find an interior angle of the regular polygon with *n* sides using the formula:

$$\frac{(n-2)\times 180^o}{n}$$

Here, n = 18

The measure of each interior angle of a regular polygon with 18 sides $= \frac{(18-2)\times 180^o}{18}$

$$= \frac{16\times 180^o}{18}$$

$$= \frac{2880^o}{18}$$

$$= 160^o$$

Step 2: Find measure of each exterior angle, subtract obtained interior angle from 180^o because each interior angle with adjacent exterior angle make angle of 180^o. So,

Measure of each exterior angle $= 180^o - 160^o$

$= 20^o$

3. Find number of sides of a regular polygon with an interior angle:

(i) 108^o

Solution: Given that interior angle is 108^o.

As we know that

$$Interior\ angle = \frac{(n-2)\times 180^o}{n}$$

Here, we have to find value of *n* (number of sides).

$$108^o = \frac{(n-2)\times 180^o}{n}$$

$$108^o n = 180^o n - 360^o$$

$$108^o n - 180^o n = -360^o$$

$$-72^o n = -360^o$$

$$n = \frac{-360^o}{-72^o}$$

$$n = 5$$

(ii) 140^o

Solution: Given that interior angle is 140^o.

As we know that

$$Interior\ angle = \frac{(n-2)\times 180^o}{n}$$

Here, we have to find value of *n* (number of sides).

$$140^o = \frac{(n-2)\times 180^o}{n}$$

$$140^o n = 180^o n - 360^o$$
$$140^o n - 180^o n = -360^o$$
$$-40^o n = -360^o$$
$$n = \frac{-360^o}{-40^o}$$
$$n = 9$$

(iii) 157.5^o

Solution: Given that interior angle is 157.5^o.

As we know that

$$Interior\ angle = \frac{(n-2)\times 180^o}{n}$$

Here, we have to find value of n (number of sides).

$$157.5^o = \frac{(n-2)\times 180^o}{n}$$
$$157.5^o n = 180^o n - 360^o$$
$$157.5^o n - 180^o n = -360^o$$
$$-22.5^o n = -360^o$$
$$n = \frac{-360^o}{-22.5^o}$$
$$n = 16$$

4. Find number of sides of a regular polygon if exterior angle is:

(i) 10^o

Solution: Given that exterior angle is 10^o.

As we know that

$$Exterior\ angle = \frac{360^o}{n}$$

Here, we have to find value of n (number of sides).

$$10^o = \frac{360^o}{n}$$
$$n = \frac{360^o}{10^o}$$
$$n = 36$$

(ii) 15^o

Solution: Given that exterior angle is 15^o.

As we know that

$$Exterior\ angle = \frac{360^o}{n}$$

Here, we have to find value of n (number of sides).

$$15^o = \frac{360^o}{n}$$

$$n = \frac{360^o}{15^o}$$

$n = 24$

(iii) 18°

Solution: Given that exterior angle is 18°.

As we know that

$$Exterior\ angle = \frac{360^o}{n}$$

Here, we have to find value of n (number of sides).

$$18^o = \frac{360^o}{n}$$

$$n = \frac{360^o}{18^o}$$

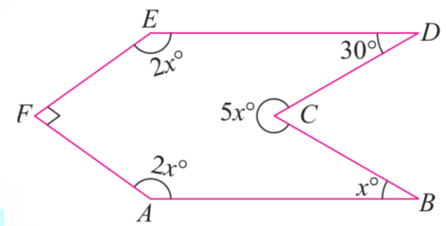
$n = 20$

5. ABCDEF is a hexagon.

(i) Find the sum of all interior angles.

(ii) Find the value of x.

(iii) Find which angle is reflex and state whether hexagon ABCDEF is concave or convex polygon.



Solution: Given figure ABCDEF is a hexagon.

(i) To find the sum of interior angles of a polygon, we use the formula:

$(n - 2) \times 180^o$

Hexagon means 6 sided polygon.

Sum of interior angles = $(6 - 2) \times 180^o$

$= 4 \times 180^o$

$= 720^o$

(ii) The sum of all interior angles = 720^o

It implies:

$2x^o + x^o + 5x^o + 30^o + 2x^o + 90^o = 720^o$

$10x^o + 120^o = 720^o$

$10x^o = 720^o - 120^o$

$10x^o = 600^o$

Divide both sides by 10

$x = 60^o$

(iii) First of all calculate all angles using value of x.

A = $2x = 2(60^o) = 120^o$, B = $x = 60^o$, C = $5x = 5(60^o) = 300^o$

D = 30^o, E = $2x = 2(60^o) = 120^o$, F = 90^o

Angle which is greater than 180^o and less than 360^o is called reflex angle.

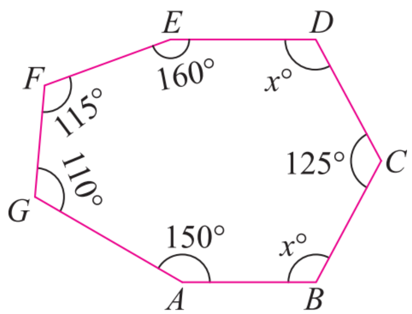
It means, 'C' is refelx angle.

The hexagon ABCDEF is concave because one of its anglea is of measure greater than 180^o.

6. ABCDEFG is a heptagon with $m\overline{BC} = m\overline{CD}$. Find:

(i) The value of x.

(ii) $m\angle ABD$



Solution: Given figure ABCDEFG is a heptagon.

(i) To find the value of x, first of all find the sum of interior angles of the polygon using the formula:

$$(n-2) \times 180^o$$

Heptagon means 7 sided polygon.

Sum of interior angles $= (7-2) \times 180^o$

$= 5 \times 180^o$

$= 900^o$

The sum of all interior angles $= 900^o$

It implies

$150^o + x^o + 125^o + x^o + 160^o + 115^o + 110^o = 900^o$

$2x^o + 660^o = 900^o$

$2x^o = 900^o - 660^o$

$2x^o = 240^o$

Divide both sides by 2

$x = 120^o$

(ii) To find $m\angle ABD$

Draw a line from B to D, it will become an isosceles triangle because $m\overline{BC} = m\overline{CD}$

In triangle, sum of all internal angles is always 180^o.

So, the measure of remaining angles will be $180^o - 125^o = 55^o$

It implies other two internal angles have measure $55^o \div 2 = 27.5^o$

We have to find $m\angle ABD$ which is outside of the angle ABC. As we already calculated

$x = 120^o$

So, $m\angle ABD = 120^o - 27.5^o = 92.5^o$

7. The interior angles of hexagon are in the ratio 1:2:3:4:5:3. Find two angles which are equal.

Solution: The sum of interior angles of a hexagon is 720^o. Let the measure of the angle be 'x'.

So according to given ratio

$x + 2x + 3x + 4x + 5x + 3x = 720^o$

$18x = 720^o$

Divide both sides by 18

$x = 40^o$

So, the angles are

$40^o, 80^o, 120^o, 160^o, 200^o, 120^o$

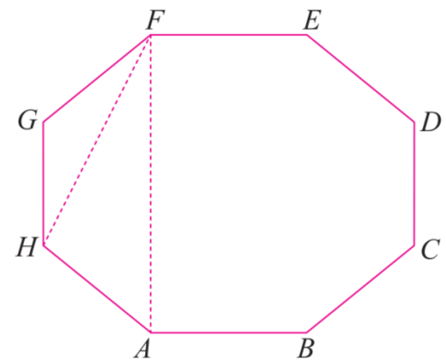
It means, two angles have same size of 120^o.

8. ABCDEFGH is a regular octagon. Find

(i) $A\hat{B}C$

(ii) $G\hat{F}H$

(iii) $A\hat{F}H$**, where AFGH is a trapezium.**



Solution: Given that ABCDEFGH is a regular octagon. It means all angles are equal in measure. So, first calculate the internal angle.

(i) Internal angle $A\hat{B}C$

The measure of each interior angle of a regular polygon with 8 sides = $\frac{(8-2)\times 180^o}{8}$

$= \frac{6\times 180^o}{8}$

$= \frac{1080^o}{8}$

$= 135^o$

It means, $A\hat{B}C = 135^o$.

(ii) To find $G\hat{F}H$

If we observe GFH is an isosceles triangle it means both corresponding angles have equal size.

In triangle GFH the angle G is 135^o.

The sum of internal angles of a triangle is always 180^o.

So, the measure of remaining angles will be $180^o - 135^o = 45^o$

It implies, $mG\hat{F}H = 45^o \div 2 = 22.5^o$

(iii) To find $A\hat{F}H$

Given that AFGH is a trapezium. In the given trapezium angle G and H are measure of 135^o. Using the property of trapezium we can say

$mA\hat{F}H + mF\hat{G}H = 180^o$

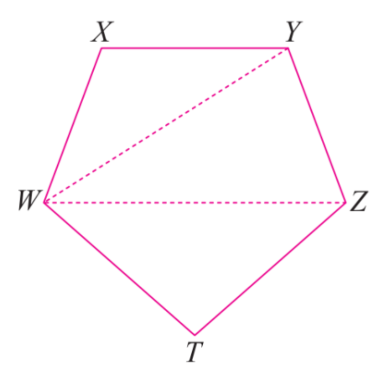
Here $mF\hat{G}H = 135^o$ so $mA\hat{F}H = 45^o$

9. WXYZT is a regular pentgon. Calculate:

(i) $W\hat{T}Z$

(ii) $T\widehat{W}Z$

(iii) $Y\widehat{W}Z$



Solution: Given that WXYZT is a regular pentagon. It means all angles are equal in measure. So, first calculate the internal angle.

(i) Internal angle $W\hat{T}Z$

The measure of each interior angle of a regular polygon with 5 sides = $\frac{(5-2)\times 180^o}{5}$

$= \frac{3 \times 180^o}{5}$

$= \frac{540^o}{5}$

$= 108^o$

It means, $W\hat{T}Z = 108^o$.

(ii) To find $T\widehat{W}Z$

If we observe TWZ is an isosceles triangle it means both corresponding angles have equal size.

In triangle TWZ the angle T is 108^o.

The sum of internal angles of a triangle is always 180^o.

So, the measure of remaining angles will be $180^o - 108^o = 72^o$

It implies, $mT\widehat{W}Z = 72^o \div 2 = 36^o$

(iii) To find $Y\widehat{W}Z$

If we observe WXYZ is a trapezium. In the given trapezium angle X and Y are measure of 108^o.

Using the property of trapezium we can say

$$mW\hat{X}Y + mX\widehat{W}Z = 180^o$$

Here $mW\hat{X}Y = 108^o$ so $mX\widehat{W}Z = 72^o$ besides this the line WY bisect the angle $X\widehat{W}Z$.

So, half of angle $X\widehat{W}Z$ will be measure of the required angle $Y\widehat{W}Z$.

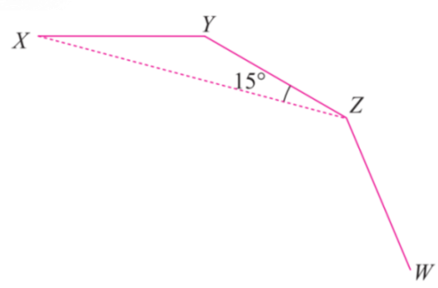
Hence, $Y\widehat{W}Z = 36^o$

10. Given that XYZW… is a regular polygon with n sides and $m\angle YZX = 15°$. Calculate:

(i) $X\hat{Y}Z$

(ii) n

(iii) $X\hat{Z}W$



Solution: Given that XYZW… is a regular polygon with n sides and $m\angle YZX = 15°$.

(i) To calculate $X\hat{Y}Z$

If we observe the shape XYZ it is an isosceles triangle. It means two sides are equal so two angles are equal. i.e.

$m\angle Z = m\angle X = 15°$

The sum of internal angles of a triangle is always 180^o.

So, the measure of remaining angle will be $180^o - 30^o = 150^o$

It implies, $mX\hat{Y}Z = 150^o$

(ii) To calculate n (number of sides)

We have found interior angle is 150^o.

As we know that

$$Interior\ angle = \frac{(n-2) \times 180^o}{n}$$

Here, we have to find value of n (number of sides).

$$150^o = \frac{(n-2)\times 180^o}{n}$$

$$150^o n = 180^o n - 360^o$$

$$150^o n - 180^o n = -360^o$$

$$-30^o n = -360^o$$

$$n = \frac{-360^o}{-30^o}$$

$$n = 12$$

(iii) To calculate $X\hat{Z}W$

We already know that each angle of this regular polygon is of measure 150^o.

It implies, $mX\hat{Z}W = 150^o - 15^o = 135^o$.

Review Exercise 13

1. Choose the correct option.

(i) An equilateral triangle has --------- equal angles.

(a) 1 (b) 2 **(c) 3** (d) 4

(ii) An isosceles triangle has --------- equal sides.

(a) 1 **(b) 2** (c) 3 (d) 4

(iii) Sum of interior angles in a triangle is equal to:

(a) 0° (b) 90° **(c) 180°** (d) 360°

(iv) Complementary angles are those whose sum is:

(a) less than 90° (b) greater than 90°
(c) equal to 90° (d) equal to 180°

(v) Vertically opposite angles are:

(a) equal (b) supplementary
(c) complementary (d) each equal to 360°

(vi) In parallel lines, ---------- angles are equal.

(a) complementary (b) supplementary
(c) interior **(d) alternate**

(vii) The sum of interior angles of parallel lines is:

(a) 0° (b) 90° **(c) 180°** (d) less than 90°

(viii) Exterior angle of a triangle is equal to sum of ----------- interior opposite angles.

(a) one **(b) two** (c) three (d) four

(ix) In square and ------------ , all sides are of equal length.

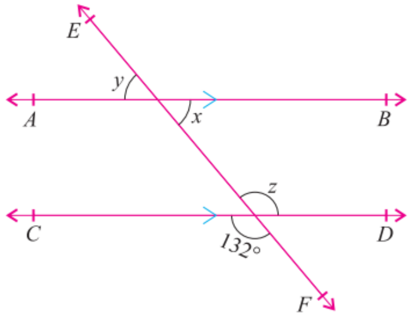
(a) rectangle (b) parallelogram **(c) rhombus** (d) Kite

(x) There is one pair of parallel sides in ---------.

(a) square (b) rectangle (c) kite **(d) trapezium**

2. Find unknowns in the following figures. Also name a pair of vertically opposite angles, corresponding angles, alternate angles and interior angles in part (iii) only.

(i)



Solution: In the given figure

$z = 132^o$ (Vertically opposite angles are equal in measure)

$z + x = 180^o$ (Sum of interior angles is equal to 180^o)

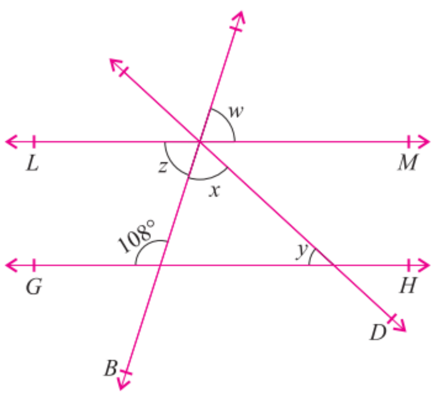
As $z = 132^o$ so,

$x = 180^o - 132^o = 48^o$

x and y are vertically opposite angles so measure of both angle will be same. So,

$y = 48^o$

(ii)



Solution: In the given figure

$z + 108^o = 180^o$ (Sum of interior angles is equal to 180^o)

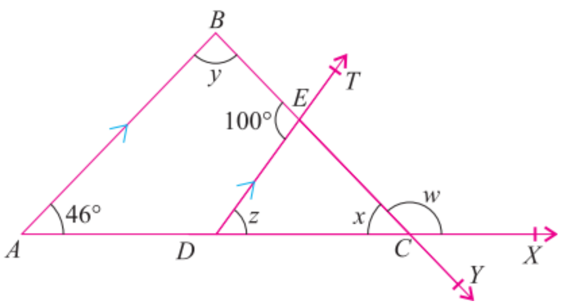
$z = 180^o - 108^o = 72^o$

$w = z = 72^o$ (Alternate angles are equal in measure)

The line CD bisect the angle 108^o it means the angle $x = 54^o$

Similarly the angle $y = 54^o$ because sum of internal angles of a triangle is equal to 180^o.

(iii)



Solution: In the given figure the ray AB is parallel to ray DT and ABCD is a quadrilateral. So,

$y + 100^o = 180^o$ (Sum of interior angles is equal to 180^o)

$y = 180^o - 100^o = 80^o$

In the isocelese triangle EDC, $m\angle E = 80^o$

It means z and x have same measure. Its implies:

$z + x + 80^o = 180^o$

$z + x = 100^o$

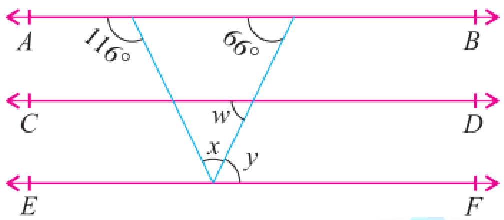
It means, $z = 50^o$, $x = 50^o$

Similarly, $w + x = 180^o$ but $x = 50^o$

So, $w = 130^o$

$\angle ECX$ and $\angle DCY$ are vertically opposite angles
$\angle DAD$ and $\angle EDC$ are corresponding angles
$\angle ABC$ and $\angle BED$ are interior angles

(iv)



Solution: In the given figure

$y = 66^o$ (Alternate angles are equal in measure)

Similarly, $x + y = 116^o$ (Alternate angles are equal in measure)

As $y = 66^o$ So,

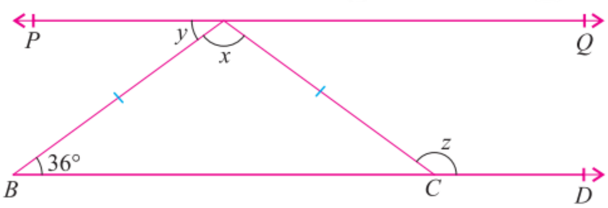
$x = 116^o - 66^o$

$x = 50^o$

The line CD bisect the angle 108^o it means the angle $x = 54^o$

Similarly, the angle $y = 54^o$ because sum of internal angles of a triangle is equal to 180^o.

(v)



Solution: In the given figure

$y = 36^o$ (Alternate angles are equal in measure)

It means, $36^o + x + 36^o = 180^o$ (Straight angle)

$x + 72^o = 180^o$

$x = 180^o - 72^o$

$x = 108^o$

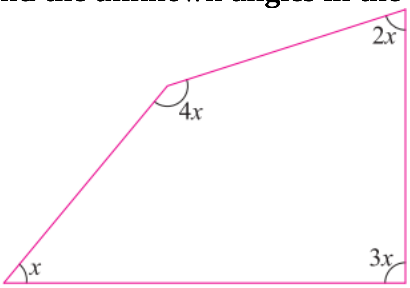
It implies, $x + y = 108^o + 36^o = 144^o$

Also, z and $x + y$ are alternate angles. It implies:

$z = 144^o$ (Alternate angles are equal in measure)

3. Find the unknown angles in the following quadrilaterals.

(i)



Solution: The given figure is a quadrilateral and sum of its internal angles equals 360^o. So,

$$x + 4x + 2x + 3x = 360^o$$

$$10x = 360^o$$

Divide both sides by 10

$$x = 36^o$$

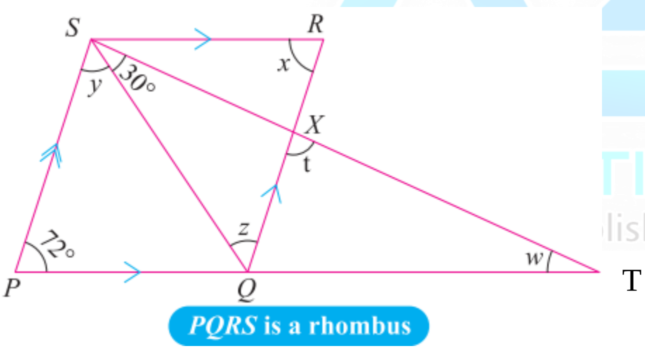
So, other angles are

$$4x = 4(36^o) = 144^o$$

$$2x = 2(36^o) = 72^o$$

$$3x = 3(36^o) = 108^o$$

(ii)



Solution: Given that figure PQRS is a rhoumbus and PST is a triangle. The sum of internal angles of a quadrilateral equals 360^o.

Opposite angles of rhombus are equal so $x = 72^o$

As two angles have measure 72^o so other two angles of rhombus have measure 108^o. The line QS bisect the angle PQR. It implies

$$z = 108^o \div 2 = 54^o$$

Here y and z are alternate angles so $y = z$ it means $y = 54^o$

Remaining angle of S is 24^o which is alternate angle with respect to w so $w = 24^o$

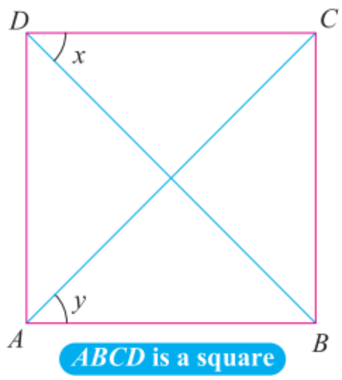
In triangle QXT, the angle XQT = $180^o - 108^o = 72^o$ and the angle QTX = w = 24^o

The sum of internal angles of a triangle is always 180^o. It implies:

The angle $t = 180^o - (72^o + 24^o)$

$t = 180^o - 96^o = 84^o$

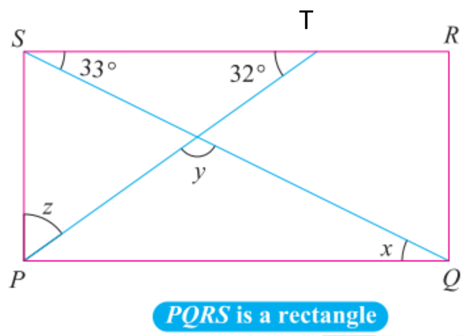
(iii)



Solution: Given that figure ABCD is a square. The sum of internal angles of a square equals 360^o. Each angle of a square have measure 90^o.

The digonal bisect the angles means $y = 45^o$ and $x = 45^o$

(iv)



Solution: Given that figure PSRQ is a rectangle. The sum of internal angles of a rectangle equals 360^o. Here the angle $x = 33^o$ because of alternate angle.

If we observe the triangle PST, the angle at vertex S = 90^o, the angle at vertex T = 32^o

So, angle P = $z = 180^o - (90^o + 32^o)$

$z = 180^o - 122^o$

$z = 58^o$

Now to find the angle y observe the below triangle.

As $z = 58^o$ so remaining angle will be $90^o - 58^o = 32^o$

So, $x + y + 32^o = 180^o$

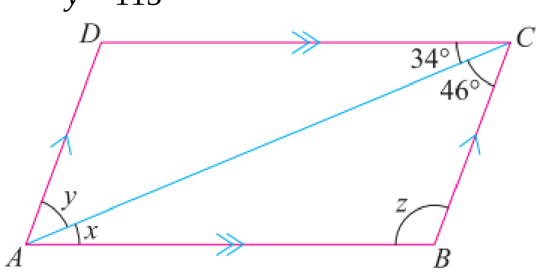
$33^o + y + 32^o = 180^o$

$65^o + y = 180^o$

$y = 180^o - 65^o$

$y = 115^o$

(v)



Solution: Given that figure ABCD is a parallelogram. Opposite angles of parallelogram are equal.

Here the angle $x = 34^o$ and $y = 46^o$ because of alternate angle.

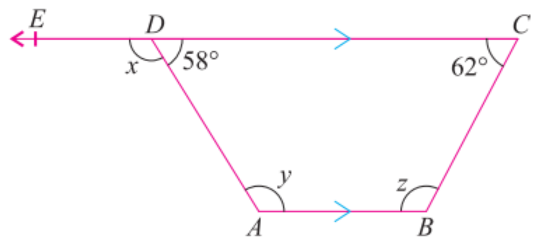
The sum of adjacent angles in parallelogram measure equales 180^o.

So, angle $z = 180^o - (34^o + 46^o)$

$z = 180^o - 80^o$

$z = 100^o$

(vi)



Solution: Given that figure ABCD is a trapezium. To find the value of x just subtract it from 180^o.

So, angle $x = 180^o - 58^o$

$x = 122^o$

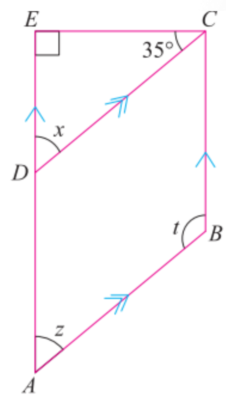
Here the measure of angle y is also 122^o because of alternate angles.

The sum of adjacent angles in trapezium measure equales 180^o.

So, angle $z = 180^o - 62^o$

$z = 118^o$

(vii)



***ABCD* is a parallelogram and *ABCE* is a trapezium**

Solution: Given that figure ABCD is a parallelogram and ABCE is a trapezium. To find the value of x use the property of sum of internal angles of a triangle.

So, angle $x = 180^o - (90^o + 35^o)$

$x = 180^o - (125^o)$

$x = 55^o$

As ABCD is a parallelogram and line AB is parallel to line DC. So, the measure of x and z are equal.

$z = 55^o$

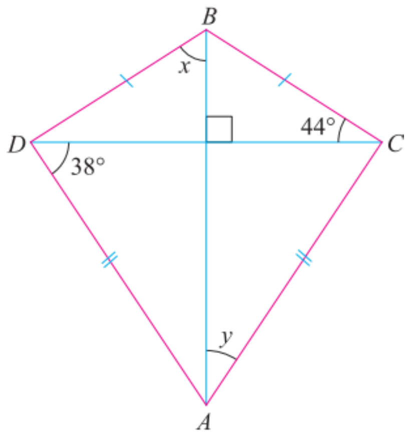
The sum of adjacent angles in parallelogram measure equales 180^o.

So, angle $t = 180^o - z$

$t = 180^o - 55^o$

$t = 125^o$

(viii)



ABCD is a kite

Solution: Given that figure ABCD is a kite. To find the value of x use the property of sum of internal angles of a triangle.

So, angle $x = 180^o - (90^o + 44^o)$

$x = 180^o - (134^o)$

$x = 46^o$ (because it is same in both triangles)

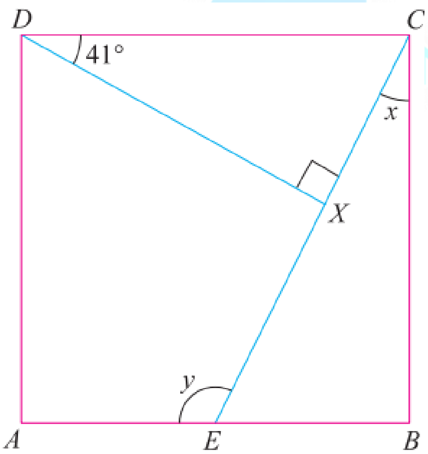
To find the value of y use the property of sum of internal angles of a triangle.

So, angle $y = 180^o - (90^o + 38^o)$

$y = 180^o - (128^o)$

$y = 52^o$ (because it is same in both triangles)

(ix)



ABCD is a square

Solution: Given that figure ABCD is a square. In right angled triangle XCD, the remaining angle at vertex C will be:

$C = 180^o - (90^o + 41^o)$

$C = 180^o - (131^o)$

$C = 49^o$

To find the value of x

$x = 90^o - 49^o$ (Each angle of a square is always equal to 90^o.)

$x = 41^o$

To find the value of y, observe the internal angles of triangle EBC. As $x = 41^o$, one angle is 90^o so the remaining angle will be measure of:

$E = 180^o - (90^o + 41^o)$

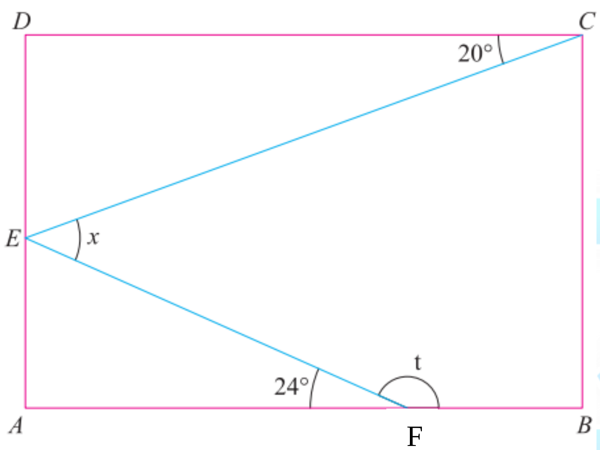
$E = 180^o - (131^o)$

$E = 49^o$

As y is the part of straight angle so subtract 49^o from 180^o.

$y = 180^o - 49^o$ (Straight angle is always equal to 180^o.)

$y = 131^o$

(x) 

ABCD is a rectangle

Solution: Given that figure ABCD is a rectangle. To find the measure of angle t:

$t = 180^o - 24^o$

$t = 156^o$

To find the value of x we have to calculate above and below angle of it.

Angle CED $= 180^o - (90^o + 20^o)$

$= 180^o - 110^o$

$= 70^o$

Angle FEA $= 180^o - (90^o + 24^o)$

$= 180^o - 114^o$

$= 66^o$

Now, to calculate x

Angle $x = 180^o - (70^o + 66^o)$

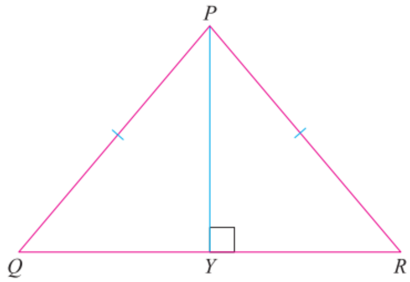
$= 180^o - 136^o$

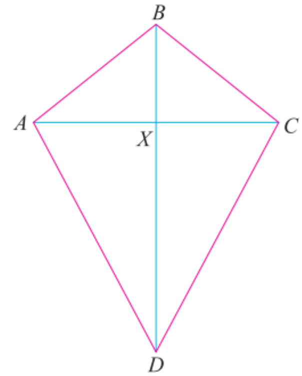
$= 44^o$

4. Given that PQR is an isosceles triangle and ABCD is a kite.

(i) Write down which lines are perpendicular.

(ii) Write down which line segment represents perpendicular distance from a point to a line.



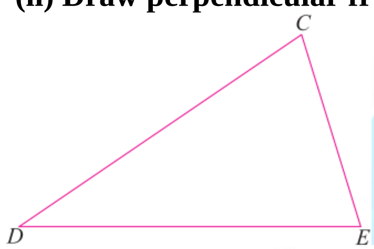


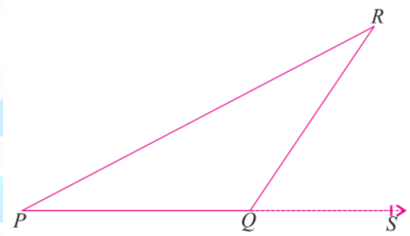
Solution: The triangle PQR is an isoscels triangle and ABCD is kite.

(i) In triangle the line PY is perpendicular on line QR. In kite ABCD the line BX is perpendicular on the line AC.

(ii) In triangle the line PY is showing perpendicular distance from the point 'P' to line QR. In kite ABCD the line BX is showing perpendicular distance from the point 'B' to the line AC.

5. (i) Draw perpendicular from a point T to a line segment AB of measure 8 cm.
(ii) Draw perpendicular from C to $\overline{DE}$ in ΔCDE and from R to $\overrightarrow{PS}$ in ΔPQR.





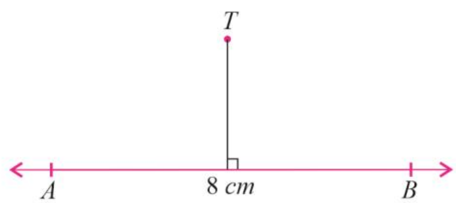
Solution: (i) To draw a perpendicular from a point on a line follow the mentioned steps.

Step 1: Using compass draw an arc whose center is point T which intersect the line AB at point F and G.

Step 2: Draw two arcs with center at point F and Point G of suitable radius intersecting at point T.

Step 3: Draw a ray from C passing through the intersecting arcs.

Hence the line from point T to the line AB is perpendicular on the line AB.



(ii) Similarly we can draw perpendicular from C to $\overline{DE}$ in ΔCDE and from R to $\overrightarrow{PS}$ in ΔPQR. See the following figures.