Unit 8 Algebraic Expressions

Exercise 8.1

1. Identify which are polynomials and which are not polynomials from the following algebraic expressions.

(i) $\sqrt{x}+y+1$

Solution: Not a polynomial because power of 'x' is not a positive integer. As $\sqrt{x}=x^{1/2}$

(ii) x^2+2x+5

Solution: It is a polynomial because powers of all variables are positive integers.

(iii) x^2+xy+z

Solution: It is a polynomial because powers of all variables are positive integers.

(iv) $y^2\sqrt{y}+z+1$

Solution: Not a polynomial because power of 'y' is not a positive integer. As $\sqrt{y}=y^{1/2}$

(v) $\sqrt{xy}+1$

Solution: Not a polynomial because power of variables 'xy' is not a positive integer. As $\sqrt{xy}=xy^{1/2}$

(vi) $7x^3+x+1$

Solution: It is a polynomial because powers of all variables are positive integers.

(vii) $\sqrt{x}+x^2+1$

Solution: Not a polynomial because power of 'x' is not a positive integer. As $\sqrt{x}=x^{1/2}$

(viii) x^4+2

Solution: It is a polynomial because powers of all variables are positive integers.

(ix) 5

Solution: It is also a polynomial because we can express it as

$5\times x^0 \qquad \because\ x^0=1$

2. Choose monomials, binomials and trinomials from the following polynomials.

(i) xy

Solution: This expression contains only 1 term which is xy. So, it is a monomial.

(ii) xyz

Solution: This expression contains only 1 term which is xyz. So, it is a monomial.

(iii) $2x+3y$

Solution: This expression contains 2 terms which are $2x$ and $3y$. So, it is a binomial.

(iv) $2x^2+1$

Solution: This expression contains 2 terms which are $2x^2$ and 1. So, it is a binomial.

(v) x^2+3x+1

Solution: This expression contains 3 terms which are x^2, $3x$ and 1. So, it is a trinomial.

(vi) $x^3 + 3$

Solution: This expression contains 2 terms which are x^3 and 3. So, it is a binomial.

(vii) $xz + y$

Solution: This expression contains 2 terms which are xz and y. So, it is a binomial.

(viii) $4yt + x$

Solution: This expression contains 2 terms which are $4yt$ and x. So, it is a binomial.

(ix) 4

Solution: This expression contains only 1 term which is 4. So, it is a monomial.

3. Identify open and close sentences from the following sentences.

(i) $x + 9 = 13$

Solution: It is an open sentence because here 'x' is the variable and we cannot easily say whether it is true or false.

(ii) $3 + 5 = 12$

Solution: It is a closed sentence because we can easily see that it is false.

(iii) $7 + 9 = 11$

Solution: It is a closed sentence because we can easily see that it is false.

(iv) $x^2 - 5 = 11$

Solution: It is an open sentence because here 'x' is the variable and we cannot easily say whether it is true or false.

(v) $3x^2 + 5x = 2$

Solution: It is an open sentence because here 'x' is the variable and we cannot easily say whether it is true or false.

(vi) $x^3 = -2$

Solution: It is an open sentence because here 'x' is the variable and we cannot easily say whether it is true or false.

(vii) $2 + 3 = 5$

Solution: It is a closed sentence because we can easily see that it is true.

(viii) $8 + 5 = 13$

Solution: It is a closed sentence because we can easily see that it is true.

(ix) $11 + 22 = 33$

Solution: It is a closed sentence because we can easily see that it is true.

Exercise 8.2

1. Add the following polynomials.

(i) $2x^2 + 3x + 5,\ 4x^2 + 7x + 9,\ 8x^2 + 7$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 2x^2 + 3x + 5 + 4x^2 + 7x + 9 + 8x^2 + 7$

Step 2: Join like terms of given polynomials.

$= 2x^2 + 4x^2 + 8x^2 + 3x + 7x + 5 + 9 + 7$

Step 3: Add coefficients and write variables as it is.

$= (2 + 4 + 8)\,x^2 + (3 + 7)\,x + (5 + 9 + 7)$

$= 14\,x^2 + 10\,x + 21$

(ii) $2x^2y + 3xy + 1,\ 4x^2y + 7xy + 9$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 2x^2y + 3xy + 1 + 4x^2y + 7xy + 9$

Step 2: Join like terms of given polynomials.

$= 2x^2y + 4x^2y + 3xy + 7xy + 1 + 9$

Step 3: Add coefficients and write variables as it is.

$= (2 + 4)\,x^2y + (3 + 7)\,xy + (1 + 9)$

$= 6\,x^2y + 10\,xy + 10$

(iii) $7x^3y^2 + 8x^2y + 3$, $5x^3y^2 + 7x^2y + 4$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 7x^3y^2 + 8x^2y + 3 + 5x^3y^2 + 7x^2y + 4$

Step 2: Join like terms of given polynomials.

$= 7x^3y^2 + 5x^3y^2 + 8x^2y + 7x^2y + 3 + 4$

Step 3: Add coefficients and write variables as it is.

$= (7 + 5)\, x^3y^2 + (8 + 7)\, x^2y + (3 + 4)$

$= 12\, x^3y^2 + 15\, x^2y + 7$

(iv) $8x^5 + 7x^2 + 1$, $7x^5 + 3x^2 + 5$, $8x^2 + 3$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 8x^5 + 7x^2 + 1 + 7x^5 + 3x^2 + 5 + 8x^2 + 3$

Step 2: Join like terms of given polynomials.

$= 8x^5 + 7x^5 + 7x^2 + 3x^2 + 8x^2 + 1 + 5 + 3$

Step 3: Add coefficients and write variables as it is.

$= (8 + 7)\, x^5 + (7 + 3 + 8)\, x^2 + (1 + 5 + 3)$

$= 15\, x^5 + 18\, x^2 + 9$

(v) $7x^2y^2 + 8xy + 1$, $3x^2y^2 + 9xy + 10$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 7x^2y^2 + 8xy + 1 + 3x^2y^2 + 9xy + 10$

Step 2: Join like terms of given polynomials.

$= 7x^2y^2 + 3x^2y^2 + 8xy + 9xy + 1 + 10$

Step 3: Add coefficients and write variables as it is.

$= (7 + 3)\, x^2y^2 + (8 + 9)\, xy + (1 + 10)$

$= 10\, x^2y^2 + 17\, xy + 11$

(vi) $8x^7 + 3x^2 - 1$, $2x^2 + 7$, $3x^7 - 9$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 8x^7 + 3x^2 - 1 + 2x^2 + 7 + 3x^7 - 9$

Step 2: Join like terms of given polynomials.

$= 8x^7 + 3x^7 + 3x^2 + 2x^2 - 1 + 7 - 9$

Step 3: Add coefficients and write variables as it is.

$= (8 + 3)\, x^7 + (3 + 2)\, x^2 + (-1 + 7 - 9)$

$= 11x^7 + 5x^2 - 3$

2. Subtract the second polynomial from the first polynomial.

(i) $8x^5 + 7x^4 + 3x^3 + 1$, $2x^5 + 3x^4 + 2x^3 + 1$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (8x^5 + 7x^4 + 3x^3 + 1) - (2x^5 + 3x^4 + 2x^3 + 1)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 8x^5 + 7x^4 + 3x^3 + 1 - 2x^5 - 3x^4 - 2x^3 - 1$

Step 3: Join like terms of given polynomials.

$= 8x^5 - 2x^5 + 7x^4 - 3x^4 + 3x^3 - 2x^3 + 1 - 1$

Step 4: Subtract coefficients and write variables as it is.

$= (8 - 2)\, x^5 + (7 - 3)\, x^4 + (3 - 2)\, x^3 + 0$

$= 6\, x^5 + 4\, x^4 + x^3$

(ii) $12x^2y - 3xy + 7y + 12$, $2x^2y + xy + 5y + 7$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (12x^2y - 3xy + 7y + 12) - (2x^2y + xy + 5y + 7)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 12x^2y - 3xy + 7y + 12 - 2x^2y - xy - 5y - 7$

Step 3: Join like terms of given polynomials.

$= 12x^2y - 2x^2y - 3xy - xy + 7y - 5y + 12 - 7$

Step 4: Subtract coefficients and write variables as it is.

$= (12 - 2)\, x^2y + (-3 - 1)\, xy + (7 - 5)\, y + 5$

$= 10\, x^2y - 4xy + 2y + 5$

(iii) $3x^2y^2 + 4xy^2 + 7x + 9$, $x^2y^2 + xy^2 - x + 1$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (3x^2y^2 + 4xy^2 + 7x + 9) - (x^2y^2 + xy^2 - x + 1)$

(iv) $8x^{10} - 12$, $7x^{10} + 9$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (8x^{10} - 12) - (7x^{10} + 9)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 3x^2y^2 + 4xy^2 + 7x + 9 - x^2y^2 - xy^2 + x - 1$

Step 3: Join like terms of given polynomials.

$= 3x^2y^2 - x^2y^2 + 4xy^2 - xy^2 + 7x + x + 9 - 1$

Step 4: Subtract coefficients and write variables as it is.

$= (3 - 1)\, x^2y^2 + (4 - 1)\, xy^2 + (7 + 1)\, x + 8$

$= 2\, x^2y^2 + 3xy^2 + 8x + 8$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 8x^{10} - 12 - 7x^{10} - 9$

Step 3: Join like terms of given polynomials.

$= 8x^{10} - 7x^{10} - 12 - 9$

Step 4: Subtract coefficients and write variables as it is.

$= (8 - 7)\, x^{10} - 12 - 9$

$= x^{10} - 21$

(v) $\mathbf{10x^5 + 4x^4 - 3x^2y + 5xy + 9,\ x^4 + x^2y + 3}$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (10x^5 + 4x^4 - 3x^2y + 5xy + 9) - (x^4 + x^2y + 3)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 10x^5 + 4x^4 - 3x^2y + 5xy + 9 - x^4 - x^2y - 3$

Step 3: Join like terms of given polynomials.

$= 10x^5 + 4x^4 - x^4 - 3x^2y - x^2y + 5xy + 9 - 3$

Step 4: Subtract coefficients and write variables as it is.

$= 10x^5 + (4 - 1)\, x^4 + (-3 - 1)\, x^2y + 5xy + 6$

$= 10x^5 + 3x^4 - 4x^2y + 5xy + 6$

(vi) $\mathbf{4x^3 - 3x^2 + 13,\ -2x^3 + x^2 + 1}$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (4x^3 - 3x^2 + 13) - (-2x^3 + x^2 + 1)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 4x^3 - 3x^2 + 13 + 2x^3 - x^2 - 1$

Step 3: Join like terms of given polynomials.

$= 4x^3 + 2x^3 - 3x^2 - x^2 + 13 - 1$

Step 4: Subtract coefficients and write variables as it is.

$= (4 + 2)\, x^3 + (-3 - 1)\, x^2 + (13 - 1)$

$= 6\, x^3 - 4x^2 + 12$

Exercise 8.3

1. Multiply the following monomials with monomials.

(i) $\mathbf{3x^2y^2,\ 7xy^3}$

Solution: To multiply monomials with monomials:

Step 1: Multiply coefficient with coefficient and same variable with same variable.

$= (3 \times 7)\, (x^2 \times x)\, (y^2 \times y^3)$

Step 2: When we multiply variables, powers of like variables are added.

$= 21x^{2+1}\, y^{2+3}$

$= 21\, x^3y^5$

(ii) $\mathbf{7xy,\ 8x^2t^2}$

Solution: To multiply monomials with monomials:

Step 1: Multiply coefficient with coefficient and same variable with same variable.

$= (7 \times 8)\, (x \times x^2)\, y\, t^2$

Step 2: When we multiply variables, powers of like variables are added.

$= 56x^{1+2}yt^2$

$= 56x^3yt^2$

(iii) $\mathbf{3x^5,\ 9x^8}$

Solution: To multiply monomials with monomials:

Step 1: Multiply coefficient with coefficient and same variable with same variable.

$= (3 \times 9)\, (x^5 \times x^8)$

Step 2: When we multiply variables, powers of like variables are added.

$= 27x^{5+8}$

$= 27\, x^{13}$

(iv) $\mathbf{xt,\ 3x^2t}$

Solution: To multiply monomials with monomials:

Step 1: Multiply coefficient with coefficient and same variable with same variable.

$= (1 \times 3)\, (x \times x^2)(\, t \times t)$

Step 2: When we multiply variables, powers of like variables are added.

$= 3x^{1+2}t^{1+1}$

$= 3x^3t^2$

(v) x^9, x^2y^2

Solution: To multiply monomials with monomials:

Step 1: Multiply coefficient with coefficient and same variable with same variable.

$= (1 \times 1)(x^9 \times x^2)\, y^2$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{9+2}\, y^2$

$= x^{11}y^2$

(vi) $7xy$, $3xy^2$

Solution: To multiply monomials with monomials:

Step 1: Multiply coefficient with coefficient and same variable with same variable.

$= (7 \times 3)(x \times x)(y \times y^2)$

Step 2: When we multiply variables, powers of like variables are added.

$= 21x^{1+1}\, y^{1+2}$

$= 21\, x^2y^3$

2. Multiply the following monomials with binomial /trinomials.

(i) x^2, $(x - 1)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (x^2 \times x) - (x^2 \times 1)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{2+1} - x^2$

$= x^3 - x^2$

(ii) x^3, $(x^2 + t^2)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (x^3 \times x^2) + (x^2 \times t^2)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{3+2} + x^2t^2$

$= x^5 + x^2\, t^2$

(iii) $x^2t\,(x^2t + y^2)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (x^2t \times x^2t) + (x^2t \times y^2)$

$= (x^2 \times x^2)(t \times t) + x^2ty^2$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{2+2}\, t^{1+1} + x^2ty^2$

$= x^4\, t^2 + x^2ty^2$

(iv) $txy\,(xy + yt)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (txy \times xy) + (txy \times yt)$

$= t(x \times x)(y \times y) + (t \times t)(y \times y)x$

Step 2: When we multiply variables, powers of like variables are added.

$= tx^{1+1}\, y^{1+1} + t^{1+1}y^{1+1}\, x$

$= tx^2y^2 + t^2y^2x$

(v) t, $(x + y + t)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (t \times x) + (t \times y) + (t \times t)$

Step 2: When we multiply variables, powers of like variables are added.

$= tx + ty + t^{1+1}$

$= tx + ty + t^2$

(vi) x^3, $(x^2y^2 + t^2y^2 + z^2)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (x^3 \times x^2y^2) + (x^3 \times t^2y^2) + (x^3 \times z^2)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{3+2}\, y^2 + x^3t^2y^2 + x^3z^2$

$= x^5y^2 + x^3t^2y^2 + x^3z^2$

(vii) $x^2yt\,(x^2y + y^2x)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (x^2yt \times x^2y) + (x^2yt \times y^2x)$

$= (x^2 \times x^2)(y \times y)t + (x^2 \times x)(y \times y^2)t$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{2+2}\, y^{1+1}\, t + x^{2+1}y^{1+2}t$

$= x^4y^2\, t + x^3y^3t$

(viii) x^2 , $(t + y^2x)$

Solution: To multiply monomial with binomial or trinomial:

Step 1: Multiply monomial with each term of binomial or trinomial.

$= (x^2 \times t) + (x^2 \times y^2x)$

$= (x^2 \times t) + (x^2 \times x)(y^2)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^2t + x^{2+1}y^2$

$= x^2t + x^3y^2$

3. Multiply the following binomials with binomials/trinomials.

(i) $(x^2 + y), (x^2 - y)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= x^2\,(x^2 - y) + y\,(x^2 - y)$

$= (x^2 \times x^2) - (x^2 \times y) + (y \times x^2) - (y \times y)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{2+2} - x^2y + yx^2 - y^{1+1}$

$= x^4 - y^2$

(ii) $(x^2 + t), (y^2 + xt)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= x^2\,(y^2 + xt) + t\,(y^2 + xt)$

$= (x^2 \times y^2) + (x^2 \times xt) + (t \times y^2) + (t \times xt)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^2y^2 + x^{2+1}t + ty^2 + xt^{1+1}$

$= x^2y^2 + x^3t + ty^2 + xt^2$

(iii) $(x^2t + y), (3x^2 + t^2 + y)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= x^2t(3x^2 + t^2 + y) + y\,(3x^2 + t^2 + y)$

$= \{(x^2t \times 3x^2) + (x^2t \times t^2) + (x^2t \times y)\} + \{(y \times 3x^2) + (y \times t^2) + (y \times y)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{(3x^{2+2}\, t) + (x^2t^{1+2}) + (x^2ty)\} + \{(3yx^2) + (yt^2) + (y^{1+1})\}$

$= 3x^4\, t + x^2t^3 + x^2ty + 3yx^2 + yt^2 + y^2$

(iv) $(x + y), (x^2 + t + y)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= x(x^2 + t + y) + y\,(x^2 + t + y)$

$= \{(x \times x^2) + (x \times t) + (x \times y)\} + \{(y \times x^2) + (y \times t) + (y \times y)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+2} + xt + xy\} + \{yx^2 + yt + y^{1+1}\}$

$= x^3 + xt + xy + yx^2 + yt + y^2$

(v) $(x^2 + x), (x^3 - x + 3)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= x^2(x^3 - x + 3) + x(x^3 - x + 3)$

$= \{(x^2 \times x^3) - (x^2 \times x) + (x^2 \times 3)\} + \{(x \times x^3) - (x \times x) + (x \times 3)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{2+3} - x^{2+1} + 3x^2\} + \{x^{1+3} - x^{1+1} + 3x\}$

$= x^5 - x^3 + 3x^2 + x^4 - x^2 + 3x$

Step 3: Join same terms and simplify.

$= x^5 + x^4 - x^3 + 3x^2 - x^2 + 3x$

$= x^5 + x^4 - x^3 + 2x^2 + 3x$

(vi) $(x + 1), (x^2 + x - 1)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= x(x^2 + x - 1) + 1(x^2 + x - 1)$

$= \{(x \times x^2) + (x \times x) - (x \times 1)\} + \{(1 \times x^2) + (1 \times x) - (1 \times 1)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+2} + x^{1+1} - x\} + \{x^2 + x - 1\}$

$= x^3 + x^2 - x + x^2 + x - 1$

Step 3: Join same terms and simplify.

$= x^3 + x^2 + x^2 - x + x - 1$

$= x^3 + 2x^2 - 1$

(vii) $(7x^2 + 1), (x^3 + x + 3)$

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= 7x^2(x^3 + x + 3) + 1(x^3 + x + 3)$

$= \{(7x^2 \times x^3) + (7x^2 \times x) + (7x^2 \times 3)\} + \{(1 \times x^3) + (1 \times x) + (1 \times 3)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{7x^{2+3} + 7x^{2+1} + 21x^2\} + \{x^3 + x + 3\}$

$= 7x^5 + 7x^3 + 21x^2 + x^3 + x + 3$

Step 3: Join same terms and simplify.

$= 7x^5 + 7x^3 + x^3 + 21x^2 + x + 3$

$= 7x^5 + 8x^3 + 21x^2 + x + 3$

(viii) $(t^2 + y^2 + x^2), (x + 1)$

Solution: To multiply trinomial with binomial:

Step 1: Multiply each term of trinomial with each term of binomial.

$= t^2(x + 1) + y^2(x + 1) + x^2(x + 1)$

$= \{(t^2 \times x) + (t^2 \times 1)\} + \{(y^2 \times x) + (y^2 \times 1)\} + \{(x^2 \times x) + (x^2 \times 1)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= t^2x + t^2 + y^2x + y^2 + x^{2+1} + x^2$

$= t^2x + t^2 + y^2x + y^2 + x^3 + x^2$

(ix) **$(9xy + 1), (x^2 - x + 2)$**

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= 9xy\,(x^2 - x + 2) + 1\,(x^2 - x + 2)$

$= \{(9xy \times x^2) - (9xy \times x) + (9xy \times 2)\} + \{(1 \times x^2) - (1 \times x) + (1 \times 2)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{9x^{1+2}y - 9x^{1+1}y + 18xy\} + \{x^2 - x + 2\}$

$= 9x^3y - 9x^2y + 18xy + x^2 - x + 2$

(x) **$(xy + yt), (xy + yt + tx)$**

Solution: To multiply binomial with binomial or trinomial:

Step 1: Multiply each term of binomial with each term of binomial or trinomial.

$= xy\,(xy + yt + tx) + yt\,(xy + yt + tx)$

$= \{(xy \times xy) + (xy \times yt) + (xy \times tx)\} + \{(yt \times xy) + (yt \times yt) + (yt \times tx)\}$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+1}\,y^{1+1} + xy^{1+1}t + x^{1+1}yt\} + \{y^{1+1}tx + y^{1+1}t^{1+1} + y\,t^{1+1}x\}$

$= x^2y^2 + xy^2t + x^2yt + y^2tx + y^2t^2 + yt^2x$

Exercise 8.4

1. Simplify the following algebraic expressions.

(i) **$2(7x + 5y) + 3(5x - 2y)$**

Solution: To simplify the given algebraic expression:

Step 1: Multiply constant terms with internal binomial. Constants will be multiplied with coefficients.

$= 2(7x + 5y) + 3(5x - 2y)$

$= (2 \times 7x) + (2 \times 5y) + (3 \times 5x) - (3 \times 2y)$

$= 14x + 10y + 15x - 6y$

Step 2: Join like terms and apply operation (addition or subtraction) on like terms.

$= 14x + 15x + 10y - 6y$

$= 29x + 4y$

(ii) **$3(x^2 + y^2) + 5(y^2 - x^2)$**

Solution: To simplify the given algebraic expression:

Step 1: Multiply constant terms with internal binomial. Constants will be multiplied with coefficients.

$= 3(x^2 + y^2) + 5(y^2 - x^2)$

$= (3 \times x^2) + (3 \times y^2) + (5 \times y^2) - (5 \times x^2)$

$= 3x^2 + 3y^2 + 5y^2 - 5x^2$

Step 2: Join like terms and apply operation (addition or subtraction) on like terms.

$= 3x^2 - 5x^2 + 3y^2 + 5y^2$

$= -2x^2 + 8y^2$

(iii) $\mathbf{7(xy + yt) - 5(yt - xy)}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply constant terms with internal binomial. Constants will be multiplied with coefficients.

$= 7(xy + yt) - 5(yt - xy)$

$= (7 \times xy) + (7 \times yt) - (5 \times yt) + (5 \times xy)$ $\quad \because \; (-) \times (-) = (+)$

$= 7xy + 7yt - 5yt + 5xy$

Step 2: Join like terms and apply operation (addition or subtraction) on like terms.

$= 7xy + 5xy + 7yt - 5yt$

$= 12xy + 2yt$

(iv) $\mathbf{(x+ y)(x + y) + (x - y)(x - y)}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply binomial with binomial. Powers will be added in case of multiplication.

$= (x + y)(x + y) + (x - y)(x - y)$

$= \{(x \times x) + (x \times y) + (y \times x) + (y \times y)\} + \{(x \times x) - (x \times y) - (y \times x) + (y \times y)\}$ $\quad \because \; (-) \times (-) = (+)$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+1} + xy + yx + y^{1+1}\} + \{x^{1+1} - xy - yx + y^{1+1}\}$

$= \{x^2 + xy + xy + y^2\} + \{x^2 - xy - xy + y^2\}$ $\quad \because \; xy = yx$

Step 3: Join like terms and apply operation (addition or subtraction) on like terms.

$= \{x^2 + 2xy + y^2\} + \{x^2 - 2xy + y^2\}$

Now again add like terms

$= x^2 + x^2 + 2xy - 2xy + y^2 + y^2$ $\quad \because$ $2xy$ and $-2xy$ cancel each other out

$= 2x^2 + 2y^2$

(v) $\mathbf{(x+ y)(x + y) - (x - y)(x - y)}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply binomial with binomial. Powers will be added in case of multiplication.

$= (x + y)(x + y) - (x - y)(x - y)$

$= \{(x \times x) + (x \times y) + (y \times x) + (y \times y)\} - \{(x \times x) - (x \times y) - (y \times x) + (y \times y)\}$ $\quad \because \; (-) \times (-) = (+)$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+1} + xy + yx + y^{1+1}\} - \{x^{1+1} - xy - yx + y^{1+1}\}$

$= \{x^2 + xy + xy + y^2\} - \{x^2 - xy - xy + y^2\}$ $\quad \because \; xy = yx$

Step 3: Join like terms and apply operation (addition or subtraction) on like terms.

$= \{x^2 + 2xy + y^2\} - \{x^2 - 2xy + y^2\}$

Subtraction will change the internal symbols of trinomial.

$= x^2 + 2xy + y^2 - x^2 + 2xy - y^2$

$= x^2 - x^2 + 2xy + 2xy + y^2 - y^2$ $\quad \because$ terms with opposite symbols cancel each other out

$= 4xy$

(vi) $\mathbf{(2x + 3y)(2x - 3y)}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply binomial with binomial. Powers will be added in case of multiplication.

$= (2x + 3y)(2x - 3y)$

$= (2x \times 2x) - (2x \times 3y) + (3y \times 2x) - (3y \times 3y)$

Step 2: When we multiply variables, powers of like variables are added.

$= \{4x^{1+1} - 6xy + 6yx - 9y^{1+1}\}$ $\qquad \because \; xy = yx$

$= 4x^2 - 9y^2$

(vii) $\boldsymbol{(x + y)(x^2 - xy + y^2)}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply binomial with trinomial. Powers will be added in case of multiplication.

$= (x + y)(x^2 - xy + y^2)$

$= (x \times x^2) - (x \times xy) + (x \times y^2) + (y \times x^2) - (y \times xy) + (y \times y^2)$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+2} - x^{1+1}y + xy^2 + yx^2 - xy^{1+1} + y^{1+2}\}$ $\qquad \because \; xy = yx$

$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3$

Join like terms and apply operation (addition or subtraction) on like terms.

$= x^3 - x^2y + yx^2 + xy^2 - xy^2 + y^3$ $\qquad \because$ terms with opposite symbols cancel each other out

$= x^3 + y^3$

(viii) $\boldsymbol{(x - y)(x^2 + xy + y^2)}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply binomial with trinomial. Powers will be added in case of multiplication.

$= (x - y)(x^2 + xy + y^2)$

$= (x \times x^2) + (x \times xy) + (x \times y^2) - (y \times x^2) - (y \times xy) - (y \times y^2)$

Step 2: When we multiply variables, powers of like variables are added.

$= \{x^{1+2} + x^{1+1}y + xy^2 - yx^2 - xy^{1+1} - y^{1+2}\}$ $\qquad \because \; xy = yx$

$= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$

Join like terms and apply operation (addition or subtraction) on like terms.

$= x^3 + x^2y - yx^2 + xy^2 - xy^2 - y^3$ $\qquad \because$ terms with opposite symbols cancel each other out

$= x^3 - y^3$

(ix) $\boldsymbol{\dfrac{x^2}{y^5} \times \dfrac{y^3}{t^7} \times \dfrac{t^2}{x^3}}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply numerators and denominators of all fractions separately.

$$= \frac{x^2 \times y^3 \times t^2}{y^5 \times t^7 \times x^3}$$

Step 2 : In case of division powers of same variables are subtracted.

$= x^{2-3} \times y^{3-5} \times t^{2-7}$

$= x^{-1} \times y^{-2} \times t^{-5}$

Step 3 : Signs of powers will be changed when we shift them in the denominator.

$$= \frac{1}{y^2 \times t^5 \times x}$$

$$= \frac{1}{y^2 t^5 x}$$

(x) $\dfrac{xyt}{2xy} \times \dfrac{3x^2y^2t^2}{xyt} \times \dfrac{ty}{yx}$

Solution: To simplify the given algebraic expression:

Step 1: Multiply numerators and denominators of all fractions separately.

$$= \frac{\cancel{xyt} \times 3x^2y^2t^2 \times ty}{2xy \times \cancel{xyt} \times yx} = \frac{3x^2y^{2+1}t^{2+1}}{2x^{1+1}y^{1+1}} = \frac{3x^2y^3t^3}{2x^2y^2}$$

Step 2 : In case of division powers of same variables are subtracted.

$$= \frac{3}{2}x^{2\text{-}2} \times y^{3\text{-}2} \times t^3$$

$$= \frac{3}{2}x^0 \times y^1 \times t^3 \qquad \because x^0 = 1$$

$$= \frac{3}{2}yt^3$$

Exercise 8.5

1. Simplify the following expression by using proper identity.

(i) (2x + 3)(2x − 3)

Solution: To simplify this algebraic expression we will use identity $(a + b)(a - b) = a^2 - b^2$

$(2x + 3)(2x - 3) = (2x)^2 - (3)^2$

Applying square on both terms.

$= (2x \times 2x) - (3 \times 3)$

$= 4x^2 - 9$

(ii) (x + 7y)(x − 7y)

Solution: To simplify this algebraic expression we will use identity $(a + b)(a - b) = a^2 - b^2$

$(x + 7y)(x - 7y) = (x)^2 - (7y)^2$

Applying square on both terms.

$= (x \times x) - (7y \times 7y)$

$= x^2 - 49y^2$

(iii) (7x + 9y)(7x − 9y)

Solution: To simplify this algebraic expression we will use identity $(a + b)(a - b) = a^2 - b^2$

$(7x + 9y)(7x - 9y) = (7x)^2 - (9y)^2$

Applying square on both terms.

$= (7x \times 7x) - (9y \times 9y)$

$= 49x^2 - 81y^2$

(iv) (x + 9)(x − 9)

Solution: To simplify this algebraic expression we will use identity $(a + b)(a - b) = a^2 - b^2$

$(x + 9)(x - 9) = (x)^2 - (9)^2$

Applying square on both terms.

$= (x \times x) - (9 \times 9)$

$= x^2 - 81$

(v) (4x + 7)(4x − 7)

Solution: To simplify this algebraic expression we will use identity $(a + b)(a - b) = a^2 - b^2$

$(4x + 7)(4x - 7) = (4x)^2 - (7)^2$

Applying square on both terms.

$= (4x \times 4x) - (7 \times 7)$

$= 16x^2 - 49$

(vi) (5x − 7)(5x + 7)

Solution: To simplify this algebraic expression we will use identity $(a + b)(a - b) = a^2 - b^2$

We can change positions in case of multiplication.

$(5x + 7)(5x - 7) = (5x)^2 - (7)^2$

Applying square on both terms.

$= (5x \times 5x) - (7 \times 7)$

$= 25x^2 - 49$

2. Find the square of the following algebraic expressions by using proper identity.

(i) $(2x + 3y)$

Solution: To take square of this algebraic expression we will use identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$$

Applying square.

$$= (2x \times 2x) + 12xy + (3y \times 3y)$$
$$= 4x^{1+1} + 12xy + 9y^{1+1}$$
$$= 4x^2 + 12xy + 9y^2$$

(ii) $(5x - 7)$

Solution: To take square of this algebraic expression we will use identity $(a - b)^2 = a^2 - 2ab + b^2$

$$(5x - 7)^2 = (5x)^2 - 2(5x)(7) + (7)^2$$

Applying square.

$$= (5x \times 5x) - 70xy + (7 \times 7)$$
$$= 25x^{1+1} - 70xy + 49$$
$$= 25x^2 - 70xy + 49$$

(iii) $(x + 9)$

Solution: To take square of this algebraic expression we will use identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(x + 9)^2 = (x)^2 + 2(x)(9) + (9)^2$$

Applying square.

$$= (x \times x) + 18xy + (9 \times 9)$$
$$= x^{1+1} + 18xy + 81$$
$$= x^2 + 18xy + 81$$

(iv) $(x + 7y)$

Solution: To take square of this algebraic expression we will use identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(x + 7y)^2 = (x)^2 + 2(x)(7y) + (7y)^2$$

Applying square.

$$= (x \times x) + 14xy + (7y \times 7y)$$
$$= x^{1+1} + 14xy + 49y^{1+1}$$
$$= x^2 + 14xy + 49y^2$$

(v) $(7x - 7y)$

Solution: To take square of this algebraic expression we will use identity $(a - b)^2 = a^2 - 2ab + b^2$

$$(7x - 7y)^2 = (7x)^2 - 2(7x)(7y) + (7y)^2$$

Applying square.

$$= (7x \times 7x) - 98xy + (7y \times 7y)$$
$$= 49x^{1+1} - 98xy + 49y^{1+1}$$
$$= 49x^2 - 98xy + 49\,y^2$$

(vi) $(7x + 8)$

Solution: To take square of this algebraic expression we will use identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(7x + 8)^2 = (7x)^2 + 2(7x)(8) + (8)^2$$

Applying square.

$$= (7x \times 7x) + 112x + (8 \times 8)$$
$$= 49x^{1+1} + 112x + 64$$
$$= 49x^2 + 112x + 64$$

3. Simplify the following algebraic expressions, by using proper identities.

(i) $(x + y)^2 + (x - y)^2$

Solution: To simplify this algebraic expression we will apply $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$$(x + y)^2 + (x - y)^2 = (x)^2 + 2(x)(y) + (y)^2 + (x)^2 - 2(x)(y) + (y)^2$$

Applying square. $= x^2 + 2xy + y^2 + x^2 - 2xy + y^2$

Join like terms and apply operations (addition or subtraction) on the like terms.

$= x^2 + x^2 + 2xy - 2xy + y^2 + y^2$ $\quad\because$ terms with opposite symbols cancel each other out

$= 2x^2 + 2y^2$

(ii) $(x + y)^2 - (x - y)^2$

Solution: To simplify this algebraic expression we will apply $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$$(x + y)^2 - (x - y)^2 = \{(x)^2 + 2(x)(y) + (y)^2\} - \{(x)^2 - 2(x)(y) + (y)^2\}$$

Applying square. $= \{x^2 + 2xy + y^2\} - \{x^2 - 2xy + y^2\}$

$= \{x^2 + 2xy + y^2 - x^2 + 2xy - y^2\}$ $\quad\because$ subtraction change the internal symbols

Join like terms and apply operations (addition or subtraction) on the like terms.

$= x^2 - x^2 + 2xy + 2xy + y^2 - y^2$ $\quad\because$ terms with opposite symbols cancel each other out

$= 4xy$

(iii) $\mathbf{(2x + 3y)^2 - (2x - 3y)^2}$

Solution: To simplify this algebraic expression we will apply $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$(2x + 3y)^2 - (2x - 3y)^2 = \{(2x)^2 + 2(2x)(3y) + (3y)^2\} - \{(2x)^2 - 2(2x)(3y) + (3y)^2\}$

Applying square. $= \{4x^2 + 12xy + 9y^2\} - \{4x^2 - 12xy + 9y^2\}$

$= \{4x^2 + 12xy + 9y^2 - 4x^2 + 12xy - 9y^2\}$ $\because$ subtraction change the internal symbols

Join like terms and apply operations (addition or subtraction) on the like terms.

$= 4x^2 - 4x^2 + 12xy + 12xy + 9y^2 - 9y^2$ $\because$ terms with opposite symbols cancel each other out

$= 24xy$

(iv) $\mathbf{(7x + y)(7x - y) - (7x - y)^2}$

Solution: To simplify this algebraic expression we will apply $(a + b)(a - b) = a^2 - b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$(7x + y)(7x - y) - (7x - y)^2 = \{(7x)^2 - (y)^2\} - \{(7x)^2 - 2(7x)(y) + (y)^2\}$

Applying square. $= \{49x^2 - y^2\} - \{49x^2 - 14xy + y^2\}$

$= \{49x^2 - y^2 - 49x^2 + 14xy - y^2\}$

Join like terms and apply operations (addition or subtraction) on the like terms.

$= 49x^2 - y^2 - 49x^2 + 14xy - y^2$

$= 49x^2 - 49x^2 - y^2 - y^2 + 14xy$ $\because$ terms with opposite symbols cancel each other out

$= -2y^2 + 14xy$

(v) $\mathbf{(3x + y)(3x - y) + (3x + y)^2}$

Solution: To simplify this algebraic expression we will apply $(a + b)(a - b) = a^2 - b^2$ and $(a + b)^2 = a^2 + 2ab + b^2$

$(3x + y)(3x - y) + (3x + y)^2 = \{(3x)^2 - (y)^2\} + \{(3x)^2 + 2(3x)(y) + (y)^2\}$

Applying square. $= \{9x^2 - y^2\} + \{9x^2 + 6xy + y^2\}$

$= \{9x^2 - y^2 + 9x^2 + 6xy + y^2\}$

Join like terms and apply operations (addition or subtraction) on the like terms.

$= 9x^2 + 9x^2 + 6xy - y^2 + y^2$ $\because$ terms with opposite symbols cancel each other out

$= 18x^2 + 6xy$

(vi) $\mathbf{(x + 3y)^2 - (x - 3y)^2}$

Solution: To simplify this algebraic expression we will apply $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$(x + 3y)^2 - (x - 3y)^2 = \{(x)^2 + 2(x)(3y) + (3y)^2\} - \{(x)^2 - 2(x)(3y) + (3y)^2\}$

Applying square $= \{x^2 + 6xy + 9y^2\} - \{x^2 - 6xy + 9y^2\}$

$= \{x^2 + 6xy + 9y^2 - x^2 + 6xy - 9y^2\}$ $\because$ subtraction change the internal symbols

Join like terms and apply operations (addition or subtraction) on the like terms.

$= x^2 - x^2 + 6xy + 6xy + 9y^2 - 9y^2$ $\because$ terms with opposite symbols cancel each other out

$= 12xy$

Exercise 8.6

1. Factorize by regrouping the terms.

(i) $\mathbf{ax^2 + by^2 + bx^2 + ay^2}$

Solution: Regrouping according to x^2 and y^2

$= ax^2 + bx^2 + ay^2 + by^2$

Taking x^2 common from first two terms and y^2 from last two terms.

$= x^2(a + b) + y^2(a + b)$

Taking $(a + b)$ common.

$= (a + b)(x^2 + y^2)$

(ii) $\mathbf{xy - pq + qy - px}$

Solution: Regrouping according to y and p

$= xy + qy - px - pq$

Taking y common from first two terms and $-p$ from last two terms.

$= y(x + q) - p(x + q)$

Taking $(x + q)$ common.

$= (x + q)(y - p)$

(iii) $ab(x^2 + y^2) + xy(a^2 + b^2)$

Solution: Multiply monomial with binomial

$= abx^2 + aby^2 + xya^2 + xyb^2$

Regrouping according to bx and ay

$= abx^2 + xyb^2 + aby^2 + xya^2$

Taking bx common from first two terms and ay from last two terms.

$= bx(ax + by) + ay(by + ax)$

Taking $(ax + by)$ common.

$= (ax + by)(bx + ay)$

(iv) $a^2x + abx + ac + aby + b^2y + bc$

Solution: Regrouping according to a and b

$= a^2x + aby + ac + b^2y + abx + bc$

Taking a common from first three terms and b from last three terms.

$= a(ax + by + c) + b(by + ax + c)$

Taking $(ax + by + c)$ common.

$= (ax + by + c)(a + b)$

(v) $x^2 + yt + xt + yx$

Solution: Regrouping according to x and t

$= x^2 + yx + yt + xt$

Taking x common from first two terms and t from last two terms.

$= x(x + y) + t(y + x)$

Taking $(x + y)$ common.

$= (x + y)(x + t)$

(vi) $6xy + 6 - 4y - 9x$

Solution: Regrouping according to x and y

$= 6xy - 9x - 4y + 6$

Taking $3x$ common from first two terms and -2 from last two terms.

$= 3x(2y - 3) - 2(2y - 3)$

Taking $(2y - 3)$ common.

$= (2y - 3)(3x - 2)$

2. Factorize by taking out common terms.

(i) $mx + my + nx + ny$

Solution: Taking 'm' common from first two terms and 'n' common from last two terms.

$= m(x + y) + n(x + y)$

Taking $(x + y)$ common.

$= (x + y)(m + n)$

(ii) $m^2 + m - nm - n$

Solution: Taking 'm' common from first two terms and '$-n$' common from last two terms.

$= m(m + 1) - n(m + 1)$

Taking $(m + 1)$ common.

$= (m + 1)(m - n)$

(iii) $3xy + x - 12yt - 4t$

Solution: Taking 'x' common from first two terms and '$-4t$' common from last two terms.

$= x(3y + 1) - 4t(3y + 1)$

Taking $(3y + 1)$ common.

$= (3y + 1)(x - 4t)$

(iv) $mn - 7m - 3n + 21$

Solution: Taking 'm' common from first two terms and '-3' common from last two terms.

$= m(n - 7) - 3(n - 7)$

Taking $(n - 7)$ common.

$= (n - 7)(m - 3)$

Exercise 8.7

1. Factorize the following quadratic algebraic expressions.

(i) $x^2 + 8x + 15$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 8 and product is 15. So, the numbers are 5 and 3 because $5 + 3 = 8$ and $5 \times 3 = 15$

Step 2: Split $8x$ into $5x$ and $3x$

$x^2 + 8x + 15 = x^2 + 5x + 3x + 15$

Step 3: Take 'x' common from first two terms and '3' from last two terms.

$= x (x + 5) + 3(x + 5)$

Taking $(x +5)$ common.

$= (x + 5) (x + 3)$

(ii) $x^2 + 9x + 20$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 9 and product is 20. So, the numbers are 5 and 4 because $5 + 4 = 9$ and $5 \times 4 = 20$

Step 2: Split $9x$ into $5x$ and $4x$

$x^2 + 9x + 20 = x^2 + 5x + 4x + 20$

Step 3: Take 'x' common from first two terms and '4' from last two terms.

$= x (x + 5) + 4(x + 5)$

Taking $(x +5)$ common.

$= (x + 5) (x + 4)$

(iii) $x^2 + 10x + 21$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 10 and product is 21. So, the numbers are 7 and 3 because $7 + 3 = 10$ and $7 \times 3 = 21$

Step 2: Split $10x$ into $7x$ and $3x$.

$$x^2 + 10x + 21 = x^2 + 7x + 3x + 21$$

Step 3: Take 'x' common from first two terms and '3' from last two terms.

$$= x(x + 7) + 3(x + 7)$$

Taking $(x + 7)$ common.

$$= (x + 7)(x + 3)$$

(iv) $x^2 + 6x + 8$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 6 and product is 8. So, the numbers are 2 and 4 because $2 + 4 = 6$ and $2 \times 4 = 8$

Step 2: Split $6x$ into $2x$ and $4x$.

$$x^2 + 6x + 8 = x^2 + 2x + 4x + 8$$

Step 3: Take 'x' common from first two terms and '4' from last two terms.

$$= x(x + 2) + 4(x + 2)$$

Taking $(x + 2)$ common.

$$= (x + 2)(x + 4)$$

(v) $x^2 + 4x - 21$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 4 and product is −21. So, the numbers are 7 and −3 because $7 - 3 = 4$ and $7 \times (-3) = -21$

Step 2: Split $4x$ into $7x$ and $-3x$.

$$x^2 + 4x - 21 = x^2 + 7x - 3x - 21$$

Step 3: Take 'x' common from first two terms and '−3' from last two terms.

$$= x(x + 7) - 3(x + 7)$$

Taking $(x + 7)$ common.

$$= (x + 7)(x - 3)$$

(vi) $2x^2 + 14x + 12$

Solution: To factorize quadratic algebraic expression:

Step 1: Take 2 common from the algebraic expression

$$2x^2 + 14x + 12 = 2(x^2 + 7x + 6)$$

Step 2: Find two numbers whose sum is 7 and product is 6. So the numbers are 6 and 1 because $6 + 1 = 7$ and $6 \times 1 = 6$

Step 3: Split $7x$ into x and $6x$.

$$2(x^2 + 7x + 6) = 2(x^2 + x + 6x + 6)$$

Step 4: Take 'x' common from first two terms and '6' from last two terms.

$$= 2\{x(x + 1) + 6(x + 1)\}$$

Taking $(x + 1)$ common.

$$= 2(x + 1)(x + 6)$$

2. Factorize the following algebraic expressions.

(i) $x^2 + 9xy + 14y^2$

Solution: To factorize quadratic algebraic expression with two variables:

Step 1: Find two terms whose sum is $9xy$ and product is $14x^2y^2$. So the numbers are $7xy$ and $2xy$ because $7xy + 2xy = 9xy$ and $7xy \times 2xy = 14x^2y^2$

Step 2: Split $9xy$ into $7xy$ and $2xy$.

$$x^2 + 9xy + 14y^2 = x^2 + 7xy + 2xy + 14y^2$$

Step 3: Take 'x' common from first two terms and '$2y$' from last two terms.

$$= x(x + 7y) + 2y(x + 7y)$$

Taking $(x + 7y)$ common.

$$= (x + 7y)(x + 2y)$$

(ii) $2x^2 + 9xy + 7y^2$

Solution: To factorize quadratic algebraic expression with two variables:

Step 1: Find two terms whose sum is $9xy$ and product is $14x^2y^2$. So the numbers are $7xy$ and $2xy$ because $7xy + 2xy = 9xy$ and $7xy \times 2xy = 14x^2y^2$

Step 2: Split $9xy$ into $7xy$ and $2xy$.

$$2x^2 + 9xy + 7y^2 = 2x^2 + 7xy + 2xy + 7y^2$$

Step 3: Take 'x' common from first two terms and 'y' from last two terms.

$$= x(2x + 7y) + y(2x + 7y)$$

Taking $(2x + 7y)$ common.

$$= (2x + 7y)(x + y)$$

(iii) $\mathbf{2x^2 + 11xy + 12y^2}$

Solution: To factorize quadratic algebraic expression with two variables:

Step 1: Find two terms whose sum is $11xy$ and product is $24x^2y^2$. So the numbers are $8xy$ and $3xy$ because $8xy + 3xy = 11xy$ and $8xy \times 3xy = 24x^2y^2$

Step 2: Split $11xy$ into $8xy$ and $3xy$.

$$2x^2 + 11xy + 12y^2 = 2x^2 + 8xy + 3xy + 12y^2$$

Step 3: Take '$2x$' common from first two terms and '$3y$' from last two terms.

$$= 2x(x + 4y) + 3y(x + 4y)$$

Taking $(x + 4y)$ common.

$$= (x + 4y)(2x + 3y)$$

(iv) $\mathbf{x^2 + 6xy + 9y^2}$

Solution: To factorize quadratic algebraic expression with two variables:

Step 1: Find two terms whose sum is $6xy$ and product is $9x^2y^2$. So the numbers are $3xy$ and $3xy$ because $3xy + 3xy = 6xy$ and $3xy \times 3xy = 9x^2y^2$

Step 2: Split $6xy$ into $3xy$ and $3xy$.

$$x^2 + 6xy + 9y^2 = x^2 + 3xy + 3xy + 9y^2$$

Step 3: Take 'x' common from first two terms and '$3y$' from last two terms.

$$= x(x + 3y) + 3y(x + 3y)$$

Taking $(x + 3y)$ common.

$$= (x + 3y)(x + 3y)$$

(v) $\mathbf{2x^2 + 9xy + 9y^2}$

Solution: To factorize quadratic algebraic expression with two variables:

Step 1: Find two terms whose sum is $9xy$ and product is $18x^2y^2$. So the numbers are $6xy$ and $3xy$ because $6xy + 3xy = 9xy$ and $6xy \times 3xy = 18x^2y^2$

Step 2: Split $9xy$ into $6xy$ and $3xy$.

$$2x^2 + 9xy + 9y^2 = 2x^2 + 6xy + 3xy + 9y^2$$

Step 3: Take '$2x$' common from first two terms and '$3y$' from last two terms.

$$= 2x(x + 3y) + 3y(x + 3y)$$

Taking $(x + 3y)$ common.

$$= (x + 3y)(2x + 3y)$$

(vi) $\mathbf{3x^2 + 10xy + 8y^2}$

Solution: To factorize quadratic algebraic expression with two variables:

Step 1: Find two terms whose sum is $10xy$ and product is $24x^2y^2$. So the numbers are $6xy$ and $4xy$ because $6xy + 4xy = 10xy$ and $6xy \times 4xy = 24x^2y^2$

Step 2: Split $10xy$ into $6xy$ and $4xy$.

$$3x^2 + 10xy + 8y^2 = 3x^2 + 6xy + 4xy + 8y^2$$

Step 3: Take '$3x$' common from first two terms and '$4y$' from last two terms.

$$= 3x(x + 2y) + 4y(x + 2y)$$

Taking $(x + 2y)$ common.

$$= (x + 2y)(3x + 4y)$$

Review Exercise 8

1. Choose the correct option.

(i) Which one of the algebraic expressions is polynomial?

(a) $\frac{1}{x} + x^2$ (b) $3x^2 + 5\sqrt{x}$ (c) $2x^{1/3} + 1$ $\boxed{\text{(d) } x^2 + \sqrt{2}}$

(ii) Which one of the algebraic expressions is not polynomial?

(a) $x^2 + \sqrt{2}$ (b) $2x + \sqrt{5}$ (c) $x^3 + x^2 + 1$ $\boxed{\text{(d) } \sqrt{x}}$

(iii) Which of the sentence is closed?

$\boxed{\text{(a) } 2 + 3 = 5}$ (b) $x + 3 = 4$ (c) $x^2 + 3 = 9$ (d) $x^2 = 4$

(iv) Which of the sentence is open?

(a) $3 + 5 = 12$ (b) $8 + 7 = 18$ $\boxed{\text{(c) } x + 3 = 5}$ (d) $2 + 2 = 4$

(v) The factorization of $x^2 - 1$ is:

$\boxed{\text{(a) } (x - 1)(x + 1)}$ (b) $(x + 1)(x^2 - x + 1)$ (c) $(x - 1)(x^2 - 1)$ (d) $(x - 1)(x^2 + 1)$

2. Add the following polynomials.

(i) $2x^2 + 5x + 3y^2, 3yx^2 + 7x + 9y$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 2x^2 + 5x + 3y^2 + 3yx^2 + 7x + 9y$

Step 2: Join like terms of given polynomials.

$= 2x^2 + 5x + 7x + 3y^2 + 3yx^2 + 9y$

Step 3: Add coefficients and write variables as it is.

$= 2x^2 + (5 + 7)x + 3y^2 + 3yx^2 + 9y$

$= 2x^2 + 12x + 3y^2 + 3yx^2 + 9y$

(ii) $5yx^3 + 7x^2 + 2y^2, 3yx^2 + 9x + 8$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 5yx^3 + 7x^2 + 2y^2 + 3yx^2 + 9x + 8$

Step 2: Join like terms of given polynomials.

$= 5yx^3 + 7x^2 + 2y^2 + 3yx^2 + 9x + 8$

(iii) $7x^2y + 2xy + 3y^2, 5yx^2 + 9y$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 7x^2y + 2xy + 3y^2 + 5yx^2 + 9y$

Step 2: Join like terms of given polynomials.

$= 7x^2y + 5yx^2 + 2xy + 3y^2 + 9y$

Step 3: Add coefficients and write variables as it is.

$= (7 + 5)\ x^2y + 2xy + 3y^2 + 9y$

$= 12x^2y + 2xy + 3y^2 + 9y$

(iv) $8x^5 + 4x^2 + 5y, 7x^9 + 8x^2 + y$

Solution: To add given polynomials we will use horizontal method of addition.

Step 1: Write symbol of addition (+) between given polynomials.

$= 8x^5 + 4x^2 + 5y + 7x^9 + 8x^2 + y$

Step 2: Join like terms of given polynomials.

$= 8x^5 + 4x^2 + 8x^2 + 5y + y + 7x^9$

Step 3: Add coefficients and write variables as it is.

$= 8x^5 + (4 + 8)x^2 + (5 + 1)y + 7x^9$

$= 8x^5 + 12x^2 + 6y + 7x^9$

3. Subtract the second polynomial from the first polynomial.

(i) $5xy^2 + 2x + 3y,\ 3x^2 + y$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (5xy^2 + 2x + 3y) – (3x^2 + y)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 5xy^2 + 2x + 3y – 3x^2 – y$

Step 3: Join like terms of given polynomials.

$= 5xy^2 + 2x + 3y - y - 3x^2$

Step 4: Subtract coefficients and write variables as it is.

$= 5xy^2 + 2x + (3 - 1)y - 3x^2$

$= 5xy^2 + 2x + 2y - 3x^2$

(ii) $\mathbf{7xy^3 + 8x^2 + 5y^2, 3x^3 + x^2 + y}$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (7xy^3 + 8x^2 + 5y^2) - (3x^3 + x^2 + y)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 7xy^3 + 8x^2 + 5y^2 - 3x^3 - x^2 - y$

Step 3: Join like terms of given polynomials.

$= 7xy^3 + 8x^2 - x^2 + 5y^2 - 3x^3 - y$

Step 4: Subtract coefficients and write variables as it is.

$= 7xy^3 + (8 - 1)x^2 + 5y^2 - 3x^3 - y$

$= 7xy^3 + 7x^2 + 5y^2 - 3x^3 - y$

(iii) $\mathbf{8xy + 7x^3 + 2x^2 , yx + 1}$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (8xy + 7x^3 + 2x^2) - (yx + 1)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 8xy + 7x^3 + 2x^2 - yx - 1$

Step 3: Join like terms of given polynomials.

$= 8xy - yx + 7x^3 + 2x^2 - 1$

Step 4: Subtract coefficients and write variables as it is.

$= (8 - 1)xy + 7x^3 + 2x^2 - 1$

$= 7xy + 7x^3 + 2x^2 - 1$

(iv) $\mathbf{7x^5 - 3xy^2 + 1, \ x^5 - x^2 - 2}$

Solution: To subtract given polynomials we will use horizontal method of subtraction.

Step 1: Write symbol of subtraction (–) between first and second polynomial.

$= (7x^5 - 3xy^2 + 1) - (x^5 - x^2 - 2)$

Step 2: When we apply subtraction to a polynomial, the internal symbols change.

$= 7x^5 - 3xy^2 + 1 - x^5 + x^2 + 2$

Step 3: Join like terms of given polynomials.

$= 7x^5 - x^5 - 3xy^2 + x^2 + 1 + 2$

Step 4: Subtract coefficients and write variables as it is.

$= (7 - 1)x^5 - 3xy^2 + x^2 + 3$

$= 6x^5 - 3xy^2 + x^2 + 3$

4. Evaluate the following products.

(i) $(2x^2)(3x^2 + 5y)$

Solution: To multiply monomial with binomial:

Step 1: Multiply monomial with each term of binomial.

$= 2x^2(3x^2) + 2x^2(5y)$

Step 2: When we multiply variables, powers of like variables are added.

$= 6x^{2+2} + 10x^2y$

$= 6x^4 + 10x^2y$

(ii) $xy\ (5yx^2 + 7x + 2)$

Solution: To multiply monomial with trinomial:

Step 1: Multiply monomial with each term of trinomial.

$= xy\ (5yx^2) + xy\ (7x) + xy\ (2)$

Step 2: When we multiply variables, powers of like variables are added.

$= 5x^{1+2}\ y^{1+1} + 7x^{1+1}y + 2xy$

$= 5x^3y^2 + 7x^2y + 2xy$

(iii) $x\ (x^2 + 2x + y)$

Solution: To multiply monomial with trinomial:

Step 1: Multiply monomial with each term of trinomial.

$= x(x^2) + x(2x) + x(y)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{1+2} + 2x^{1+1} + xy$

$= x^3 + 2x^2 + xy$

(iv) $(x + y)(x + y + z)$

Solution: To multiply binomial with trinomial:

Step 1: Multiply each term of binomial with each term of trinomial.

$= x(x) + x(y) + x(z) + y(x) + y(y) + y(z)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{1+1} + xy + xz + xy + y^{1+1} + yz$

$= x^2 + xy + xz + xy + y^2 + yz$

$= x^2 + 2xy + xz + y^2 + yz$

(v) $(x^2 + y), (2x + 3y)$

Solution: To multiply binomial with binomial:

Step 1: Multiply each term of binomial with each term of binomial.

$= x^2(2x) + x^2(3y) + y(2x) + y(3y)$

Step 2: When we multiply variables, powers of like variables are added.

$= 2x^{2+1} + 3x^2y + 2xy + 3y^{1+1}$

$= 2x^3 + 3x^2y + 2xy + 3y^2$

(vi) $(x^2 + y^2), (x + y)$

Solution: To multiply binomial with binomial:

Step 1: Multiply each term of binomial with each term of binomial.

$= x^2(x) + x^2(y) + y^2(x) + y^2(y)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{2+1} + x^2y + y^2x + y^{2+1}$

$= x^3 + x^2y + y^2x + y^3$

(vii) $(x - 1)\ (x^2 + x + 1)$

Solution: To multiply binomial with trinomial:

Step 1: Multiply each term of binomial with each term of trinomial.

$= x(x^2) + x(x) + x(1) - 1(x^2) - 1(x) - 1(1)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{1+2} + x^{1+1} + x - x^2 - x - 1$

$= x^3 + x^2 + x - x^2 - x - 1$

$= x^3 - 1$

(viii) $(x - 2)\ (x + 2)$

Solution: To multiply binomial with binomial:

Step 1: Multiply each term of binomial with each term of binomial.

$= x(x) + x(2) - 2(x) - 2(2)$

Step 2: When we multiply variables, powers of like variables are added.

$= x^{1+1} + 2x - 2x - 4$

$= x^2 - 4$

5. Factorize by regrouping.

(i) $mn - 4n - 11m + 44$

Solution: Rearrange the terms of polynomial

$mn - 11m - 4n + 44$

Taking 'm' common from first two terms and '-4' common from last two terms.

$= m(n - 11) - 4(n - 11)$

Taking $(n - 11)$ common.

$= (n - 11)(m - 4)$

(ii) $xy + 3x + y + 3$

Solution: Rearrange the terms of polynomial

$xy + y + 3 + 3x$

Taking 'y' common from first two terms and '3' common from last two terms.

$= y(x + 1) + 3(1 + x)$

Taking $(x + 1)$ common.

$= (x + 1)(y + 3)$

(iii) $ax^3 + bx^3 + ay^3 + by^3$

Solution: Taking 'x^3' common from first two terms and 'y^3' common from last two terms.

$= x^3(a + b) + y^3(a + b)$

Taking $(a + b)$ common.

$= (a + b)(x^3 + y^3)$

(iv) $x^2 + xy - 2x - 2y$

Solution: Rearrange the terms of polynomial

$x^2 - 2x + xy - 2y$

Taking 'x' common from first two terms and 'y' common from last two terms.

$= x(x - 2) + y(x - 2)$

Taking $(x - 2)$ common.

$= (x - 2)(x + y)$

6. Factorize quadratic expressions by splitting middle term.

(i) $4x^2 + 2x - 2$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 2 and product is –8. So, the numbers are 4 and –2 because 4 – 2 = 2 and 4 × (– 2) = – 8

Step 2: Split $2x$ into $4x$ and $-2x$.

$4x^2 + 2x - 2 = 4x^2 + 4x - 2x - 2$

Step 3: Take '$4x$' common from first two terms and '– 2' from last two terms.

$= 4x(x + 1) - 2(x + 1)$

Taking $(x + 1)$ common.

$= (x + 1)(4x - 2)$

(ii) $9x^2 + 9x - 4$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 9 and product is –36. So, the numbers are 12 and –3 because 12 – 3 = 9 and 12 × (– 3) = – 36

Step 2: Split $9x$ into $12x$ and $-3x$.

$9x^2 + 9x - 4 = 9x^2 + 12x - 3x - 4$

Step 3: Take '$3x$' common from first two terms and '– 1' from last two terms.

$= 3x(3x + 4) - 1(3x + 4)$

Taking $(3x + 4)$ common.

$= (3x + 4)(3x - 1)$

(iii) $\mathbf{x^2 + 18x + 77}$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 18 and product is 77. So, the numbers are 11 and 7 because 11 + 7 = 18 and 11 × 7 = 77

Step 2: Split $18x$ into $11x$ and $7x$.

$x^2 + 18x + 77 = x^2 + 11x + 7x + 77$

Step 3: Take 'x' common from first two terms and '7' from last two terms.

$= x(x + 11) + 7(x + 11)$

Taking $(x + 11)$ common.

$= (x + 11)(x + 7)$

(iv) $\mathbf{12x^2 + 11x - 5}$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 11 and product is –60. So, the numbers are 15 and –4 because 15 – 4 = 11 and 15 × (– 4) = – 60

Step 2: Split $11x$ into $15x$ and $-4x$.

$12x^2 + 11x - 5 = 12x^2 + 15x - 4x - 5$

Step 3: Take '$3x$' common from first two terms and '– 1' from last two terms.

$= 3x(4x + 5) - 1(4x + 5)$

Taking $(4x + 5)$ common.

$= (4x + 5)(3x - 1)$

(v) $\mathbf{28x^2 + x - 2}$

Solution: To factorize quadratic algebraic expression:

Step 1: Find two numbers whose sum is 1 and product is –56. So, the numbers are 8 and –7 because 8 – 7 = 1 and 8 × (– 7) = – 56

Step 2: Split x into $8x$ and $-7x$.

$28x^2 + x - 2 = 28x^2 + 8x - 7x - 2$

Step 3: Take '$4x$' common from first two terms and '– 1' from last two terms.

$= 4x(7x + 2) - 1(7x + 2)$

Taking $(7x + 2)$ common.

$= (7x + 2)(4x - 1)$

7. Identify open and closed sentences.

(i) $\mathbf{x + 9 = 13}$

Solution: It is an open sentence because here 'x' is the variable and we cannot easily say whether it is true or false.

(ii) $\mathbf{2x^2 + 3 = 12}$

Solution: It is an open sentence because here 'x' is the variable and we cannot easily say whether it is true or false.

(iii) **7 + 8 = 15**

Solution: It is a closed sentence because we can easily see that it is true.

(iv) **8 + 9 = 20**

Solution: It is a closed sentence because we can easily see that it is false.

(v) **2 + 3 = 23**

Solution: It is a closed sentence because we can easily see that it is false.